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NOTICE.

THE solutions of the examples numbered 7 to 12 in pages 5 and 6 of the Fourth Edition of the Manual will be found at pages 6 and 7 of this New Edition of the "Key," numbered 1 to 6 respectively.

The new examples introduced into the Fourth Edition of the Manual are—

Pages.	Examples.
5, 6	1 to 6
16	1 to 12
70 to 88	All examples in these pages.

The solutions of these new examples will be found at the end of this New Edition of the "Key," beginning at p. 72.

Page 6.

(1.) By equation (3)

$$N'' = 206265'' \times \frac{a}{r} = 206265 \times \frac{9}{100}$$

$$= 2062.65 \times 9 = 18563.85''$$

$$= 5^\circ\ 9'\ 23''.85. \quad \textit{Ans.}$$

(2.) $3'\ 28'' = 208'' = N''$

by equation (3)

$$N'' = 206265'' \times \frac{a}{r}$$

$$\therefore r = \frac{206265 \times a}{N} = \frac{206265 \times 6}{208}$$

$$= \frac{1237590}{208} = 5949.9519 \text{ ft.} = 1983.3173 \text{ yds.}$$

$$= 1 \text{ mile } 223.3173 \text{ yds.} \quad \textit{Ans.}$$

(3.) $a = 7926$ miles, $r = 237638$ miles

$\therefore$ by equation (3)

$$N'' = 206265'' \times \frac{a}{r}$$

$$= 206265'' \times \frac{7926}{237638} = \frac{1634856390}{237638}$$

$$= 6879''.608 = 1^\circ\ 54'\ 39''.608. \quad \textit{Ans.}$$

(4.) $31'\ 7'' = 1867'' = N''$, $r = 237638$ miles;

by equation (3)

$$N'' = 206265'' \times \frac{a}{r}$$

$$\therefore a = \frac{N \times r}{206265} = \frac{1867 \times 237638}{206265}$$

$$= \frac{443670146}{206265} = 2150.97 \text{ miles.} \quad \textit{Ans.}$$

(5.) $N'' = 17''.2, \; a = 7926$ miles;
by equation (3)

$$N'' = 206265'' \times \frac{a}{r}$$

$$\therefore r = \frac{206265 \times a}{N} = \frac{206265 \times 7926}{17.2}$$

$$= \frac{1634856390}{17.2} = \frac{16348563900}{172}$$

$$= 95049790.1 \text{ miles.} \quad \textit{Ans.}$$

(6) $32'\ 3'' = 1923'' = N''$,
$r = 95049790$ miles (by the answer to the last example).
By equation (3)

$$N'' = 206265'' \times \frac{a}{r},$$

$$\therefore a = \frac{N \times r}{206265} = \frac{1923 \times 95049790}{206265}$$

$$= \frac{182780746170}{206265} = 886145.2 \text{ miles.} \quad \textit{Ans.}$$

Page 9.

(3.) By equation (6)

$$\cos A = \frac{1}{\sec A}$$

and by equation (2)

$$\sec A = \sqrt{(1 + \tan^2 A)},$$

$$\therefore \cos A = \frac{1}{\sqrt{(1 + \tan^2 A)}}. \quad \textit{Ans.}$$

(4.) By equation (8)

$$\text{cosec}\,A = \frac{1}{\sin A}$$

and by example (1)

$$\sin A = \sqrt{(1 - \cos^2 A)},$$

$$\therefore\ \text{cosec}\,A = \frac{1}{\sqrt{(1 - \cos^2 A)}}. \quad \textit{Ans.}$$

(5.) By example (1)

$$\sin A = \sqrt{(1 - \cos^2 A)}$$

and by equation (6)

$$\cos A = \frac{1}{\sec A}$$

$$\therefore\ 1 - \cos^2 A = 1 - \frac{1}{\sec^2 A} = \frac{\sec^2 A - 1}{\sec^2 A}$$

$$\text{and } \therefore\ \sin A = \sqrt{(1 - \cos^2 A)} = \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}}$$

$$= \frac{\sqrt{(\sec^2 A - 1)}}{\sec A}. \quad \textit{Ans.}$$

Page 14.

(3.) $\sin 30^\circ = .50000$, and

$$\cos 30^\circ = .86602 \text{ (by page 13)}$$

$$\therefore\ \tan 30^\circ = \frac{\sin 30^\circ}{\cos 30^\circ} = \frac{50000}{86602} = .57735. \quad \textit{Ans.}$$

(6.) Let x = greater part of 1 cut in extreme and mean ratio, then $1 - x$ = less part, but product of whole and less part = square of greater part (Euclid, Book VI. p. 30),

$$\therefore\ 1 \times (1 - x) = x^2 \text{ or } x^2 + x = 1$$

$$\therefore x^2 + x + \frac{1}{4} = \frac{5}{4}$$

$$\text{and } \therefore x + \frac{1}{2} = \frac{\sqrt{5}}{2} \text{ and } x = \frac{\sqrt{5} - 1}{2}$$

(7.) $\cos 18^\circ = \sqrt{(1 - \sin^2 18^\circ)} = \sqrt{\left\{1 - \left(\frac{\sqrt{5}-1}{4}\right)^2\right\}}$
by last example

$$= \sqrt{\left(1 - \frac{6 - 2\sqrt{5}}{16}\right)} = \sqrt{\left(\frac{10 + 2\sqrt{5}}{16}\right)}$$

$$= \frac{\sqrt{(10 + 2\sqrt{5})}}{4} = \frac{\sqrt{(10 + 4.47213595)}}{4}$$

$$= \frac{3.80422}{4} = .95105. \quad \textit{Ans.}$$

(8) $\sin 18^\circ = \dfrac{\sqrt{5}-1}{4}$ and $\cos 18^\circ = \dfrac{\sqrt{(10+2\sqrt{5})}}{4}$

$$\therefore \tan 18^\circ = \frac{\sin 18^\circ}{\cos 18^\circ} = \frac{\sqrt{5}-1}{\sqrt{(10+2\sqrt{5})}}$$

$$= \frac{1.2360679977}{3.80422} = .32492. \quad \textit{Ans.}$$

Page 15.

(9) $\sin 72^\circ = \cos 18^\circ = .95105$
by example (7);
$\cos 72^\circ = \sin 18^\circ = .30901$
by example (6),

$$\therefore \tan 72^\circ = \frac{95105}{30901} = 3.07773. \quad \textit{Ans.}$$

or, by example (8)

$$\tan 76^\circ = \cot 18^\circ = \frac{1}{\tan 18^\circ}$$

$$= \frac{1.00000}{.32492} = 3.07768. \quad \textit{Ans.}$$

Page 17.

(2.) $$c = \sqrt{(a^2+b^2)} = \sqrt{(141.047^2 + 350^2)}$$
$$= \sqrt{(19995.939649 + 122500)}$$
$$= \sqrt{(142495.939649)} = 377.486$$
$$\tan A = \frac{a}{b} = \frac{141.407}{350} = \frac{282.814}{700}$$

(obtained by multiplying numerator and denominator by 2)

$$= .40402 = \tan 22^\circ,$$

by Table I. at end of Manual,

$$\therefore A = 22^\circ \text{ and } B = 90^\circ - 22^\circ = 68^\circ$$

(3.) $$c = \sqrt{(a^2+b^2)} = \sqrt{(127.38^2 + 250^2)}$$
$$= \sqrt{(16225.6644 + 62500)} = \sqrt{(78725.6644)} = 280.58$$
$$\tan A = \frac{a}{b} = \frac{127.38}{250} = \frac{509.52}{1000}$$

(by multiplying numerator and denominator by 4)

$$= .50952 = \tan 27^\circ \text{ (by Table I.)}$$
$$\therefore A = 27^\circ \text{ and } B = 90^\circ - 27^\circ = 63^\circ.$$

Page 18.

(1.) $b = \sqrt{(c^2 - a^2)} = \sqrt{(15^2 - 5.1303^2)}$

$= \sqrt{(225 - 26.31997809)} = \sqrt{(198.68002191)}$

$= 14.09539$.

$$\sin A = \frac{a}{c} = \frac{5.1303}{15} = \frac{10.2606}{30}$$

$= .34202 = \sin 20^\circ$ (by Table I.)

$\therefore A = 20^\circ$ and $B = 90^\circ - 20^\circ = 70^\circ$.

(2.) $b = \sqrt{(c^2 - a^2)} = \sqrt{(250^2 - 128.76^2)}$

$= \sqrt{(62500 - 16579.1376)} = \sqrt{(45920.8624)} = 214.29$

$$\sin A = \frac{a}{c} = \frac{128.76}{250} = \frac{515.04}{1000}$$

$= .51504 = \sin 31^\circ$ (by Table II.)

$\therefore A = 31^\circ$ and $B = 90^\circ - 31^\circ = 59^\circ$

(3.) $b = \sqrt{(c^2 - a^2)} = \sqrt{(175^2 - 141.57675^2)}$

$= \sqrt{(30625 - 20043.9761405625)}$

$= \sqrt{(10581.0238594375)} = 102.86413$

$$\sin A = \frac{a}{c} = \frac{141.57675}{175} = \frac{566.307}{700}$$

$= .80901 = \sin 54^\circ$ (by Table II.)

$\therefore A = 54^\circ$ and $B = 90^\circ - 54^\circ = 36^\circ$.

Page 19, *Case* III.

(1.) $B = 90^\circ - A = 90^\circ - 23^\circ = 67^\circ$

$$b = a \cot A = 172 \cot 23^\circ$$

$$= 172 \times 2.35585 \text{ (by Table III.)} = 405.2062$$

$$c = \frac{a}{\sin A} = \frac{172}{\sin 23^\circ} = \frac{172}{.39073}$$

$$\text{(by Table I.)} = 440.2016$$

(2.) $A = 90^\circ - B = 90^\circ - 60^\circ = 30^\circ$

$$b = a \cot A = 315 \cot 30^\circ = 315 \times 1.73205$$

$$\text{(by Table III.)} = 545.59575$$

$$c = \frac{a}{\sin A} = \frac{315}{.50000} = 630$$

(3.) $A = 90^\circ - B = 90^\circ - 57^\circ = 33^\circ$

$$b = a \cot A = 2100 \cot 33^\circ = 2100 \times 1.53986$$

$$\text{(by Table II.)} = 3233.706$$

$$c = \frac{a}{\sin A} = \frac{2100}{\sin 33^\circ} = \frac{2100}{.54464} = 3855.757.$$

Page 19, *Case* IV.

(1.) $B = 90^\circ - A = 90^\circ - 35^\circ = 55^\circ.$

$$a = c \sin A = 240 \sin 35^\circ$$

$$= 240 \times .57357 \text{ (by Table II.)} = 137.6568$$

$$b = c \cos A = 240 \cos 35^\circ$$

$$= 240 \times .81915 \text{ (by Table II.)} = 196.596$$

(2.) $A = 90^\circ - B = 90^\circ - 44^\circ = 46^\circ$

$$a = c \sin A = 575 \sin 46^\circ$$
$$= 575 \times .71934 \text{ (by Table II.)} = 413.6205$$
$$b = c \cos A = 575 \cos 46^\circ$$
$$= 575 \times .69466 \text{ (by Table II.)} = 399.4295$$

(3.) $B = 90^\circ - A = 90^\circ - 29^\circ = 61^\circ$

$$a = c \sin A = 7 \sin 29^\circ = 7 \times .48481 = 3.39367$$
$$b = c \cos A = 7 \cos 29^\circ = 7 \times .87462$$
$$\text{(by Table III.)} = 6.12234$$

Page 21.

(2.) $c = \sqrt{(a^2 + b^2)} = \sqrt{(9 + 16)} = \sqrt{25} = 5$

$\log \tan A = 10 + \log a - \log b$ (see page 20, Manual)

$$\left.\begin{aligned} 10 + \log a = 10 + \log 3 &= 10.4771213 \\ \log b = \log 4 &= .6020600 \end{aligned}\right\} \text{subtract}$$

$$\begin{aligned} \therefore \log \tan A &= 9.8750613 \\ \log \tan 36^\circ 52' &= 9.8750102 \end{aligned}$$

$$511 = \text{diff.}$$

$$\text{Tab. diff.} = 2632$$

$$\therefore \frac{511 \times 60''}{2632} = 11'' \text{ (by Rule XII. Appendix)}$$

$$\therefore A = 36^\circ 52' 11'' \text{ and } B = 90^\circ - A$$
$$= 90^\circ - 36^\circ 52' 11'' = 53^\circ 7' 49''.$$

N. B.—In this example I have used Chambers' Mathematical Tables; I shall also employ them in the subsequent examples whenever the necessary Tables are not found in the Manual.

(3.) $a = 1$ mile $= 1760$ yds.

$b = 3$ fur. 7 per. $= 698.5$ yds.

$$\log \tan A = 10 + \log a - \log b$$

$$\left.\begin{array}{r} 10 + \log a = 10 + \log 1760 = 13.2455127 \\ \log b = \log 698.5 = 2.8441664 \end{array}\right\}\text{subtract}$$

$$\begin{array}{r} \therefore \log \tan A = 10.4013463 \\ \log \tan 68^\circ 21' = 10.4012775 \\ \hline 688 = \text{diff.} \end{array}$$

Tab. diff. $= 3683$ $\therefore$ (by Rule XII. App.) $\dfrac{688 \times 60''}{3683} = 11''$

$\therefore A = 68^\circ 21' 11''$ and $B = 90^\circ - 68^\circ 21' 11''$
$= 21^\circ 38' 49''$

$\log c = 10 + \log a - \log \sin A =$

$10 + \log 1760 - \log \sin 68^\circ 21' 11''$

$$\left.\begin{array}{r} 10 + \log a = 10 + \log 1760 = 13.2455127 \\ \log \sin 68^\circ 21' = 9.9682283 \end{array}\right\}\text{subtract}$$

$$\text{Tab. diff.} = 502 \;\therefore\; \frac{502 \times 11''}{60''} = \left.\begin{array}{r} 3.2772844 \\ 92 \end{array}\right\}\begin{array}{l}\text{subtract} \\ \text{(by Rule} \\ \text{XI.)}\end{array}$$

$$\begin{array}{r} \therefore \log c = 3.2772752 \\ \log 18935 = .2772653 \\ \hline 99 = \text{diff.} \end{array} \quad \text{(omitting the characteristic)}$$

Tab. diff. $= 230$ $\therefore$ (by Rule VI. App.)

$\dfrac{99}{230} = .43$ $\therefore c = 1893.543$ yds.

The number of the digits of the integral part being *four*, since the characteristic is the *numeral three.*

(4.) $a = 144.5$ feet, $b = \frac{1}{4}$ mile $= 1320$ feet.

$$\log \tan A = 10 + \log a - \log b$$

$$\left.\begin{array}{r} 10 + \log a = 10 + \log 144.5 = 12.1598678 \\ \log b = \log 1320 = 3.1205739 \end{array}\right\} \text{subtract}$$

$$\begin{array}{r} \therefore \log \tan A = 9.0392939 \\ \log \tan 6^\circ 14' = 9.0383159 \\ \hline 9780 = \text{diff.} \end{array}$$

Tab. diff. $= 11689$ $\therefore$ (by Rule XII. App.) $\dfrac{9780 \times 60''}{11689} = 50''$

$$\therefore A = 6^\circ 14' 50'', \text{ and}$$

$$B = 90^\circ - 6^\circ 14' 50'' = 83^\circ 45' 10''.$$

$$\log c = 10 + \log a - \log \sin A$$

$$= 10 + \log a - \log \sin 6^\circ 14' 50''$$

$$\left.\begin{array}{r} 10 + \log a = 12.1598678 \\ \log \sin 6^\circ 14' = 9.0357407 \end{array}\right\} \text{subtract}$$

$$\begin{array}{l} \text{Tab. diff.} = 11551 \\ \therefore \dfrac{11551 \times 50''}{60''} = \end{array} \left.\begin{array}{r} 3.1241271 \\ \\ 9626 \end{array}\right\} \begin{array}{l} \text{subtract} \\ \text{(Rule XI.)} \end{array}$$

$$\therefore \log c = 3.1231645$$

$$\log 13279 = .1231654$$

(The characteristic being omitted, as in similar cases)

$$\therefore c = 1327.9 \text{ feet, } \textit{very nearly}.$$

(5.) $$\log \tan A = 10 + \log a - \log b$$

$$\left.\begin{array}{r} 10 + \log a = 10 + 1341 = 13.1274288 \\ \log b = \log 1432 = 3.1559430 \end{array}\right\} \text{subtract}$$

$$\begin{array}{r} \therefore \log \tan A = 9.9714858 \\ \log \tan 43^\circ 7' = 9.9714286 \\ \hline 572 = \text{diff.} \end{array}$$

Tab. diff. = 2532

$$\therefore \text{(by Rule XII. App.) } \frac{572 \times 60''}{2532} = 13''$$

$$\therefore A = 43^\circ 7' 13'' \text{ and } B = 90^\circ - 43^\circ 7' 13'' = 46^\circ 52' 47''$$

$$\log c = 10 + \log a - \log \sin A = 10 + \log a - \log \sin 43^\circ 7' 13$$

$$\left.\begin{array}{r} 10 + \log a = 13.1274288 \\ \log \sin 43^\circ 7' = 9.8347297 \end{array}\right\} \text{subtract}$$

Tab. diff. = 1349

$$\left.\begin{array}{r} 3.2926991 \\ \therefore \frac{1349 \times 13''}{60''} = \qquad 292 \end{array}\right\} \text{subtract}$$

$$\begin{array}{r} \therefore \log c = 3.2926699 \\ \log 19618 = .2926547 \\ \hline 152 = \text{diff.} \end{array}$$

$$\text{Tab. diff.} = 222 \therefore \frac{152}{222} = .7$$

$$\therefore c = 1961.87$$

(6.) $$\log \tan A = 10 + \log a - \log b$$

$$\left.\begin{array}{r} 10 + \log a = 10 + \log 1760 = 13.2455127 \\ \log b = \log 1000 = 3.0000000 \end{array}\right\} \text{subtract}$$

$$\begin{array}{r} \therefore \log \tan A = 10.2455127 \\ \log \tan 60^\circ 23' = 10.2452971 \\ \hline 2156 = \text{diff.} \end{array}$$

$$\text{Tab. diff.} = 2940 \therefore \frac{2156 \times 60''}{2940} = 44''$$

(by Rule XII. App.)

$$\therefore A = 60^\circ\, 23'\, 44'' \text{ and } B = 90^\circ - 60^\circ\, 23'\, 44''$$
$$= 29^\circ\, 36'\, 16''$$

$$c = \sqrt{(a^2 + b^2)} = \sqrt{(1760^2 + 1000^2)}$$
$$= \sqrt{(3097600 + 1000000)} = \sqrt{(4097600)} = 2024.25$$

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(2.) $$\log \sin A = 10 + \log a - \log c$$

$$\left.\begin{array}{r} 10 + \log a = 10 + \log 512 = 12.7092700 \\ \log c = \log 1007 = 3.0030295 \end{array}\right\} \text{subtract}$$

$$\begin{array}{r} \therefore \log \sin A = 9.7062405 \\ \log \sin 30^\circ\, 33' = 9.7061116 \\ \hline 1289 = \text{diff.} \end{array}$$

$$\text{Tab. diff.} = 2140 \therefore \frac{1289 \times 60''}{2140} = 36''$$

$$\therefore A = 30^\circ\, 33'\, 36'' \text{ and } B = 90^\circ - 30^\circ\, 33'\, 36''$$
$$= 59^\circ\, 26'\, 24''$$

(3.) $$\log \sin A = 10 + \log a - \log c$$

$$\left.\begin{array}{r} 10 + \log a = 10 + \log 32.712 = 11.5147071 \\ \log c = \log 96.2 = 1.9831751 \end{array}\right\} \text{subtract}$$

$$\begin{array}{r} \therefore \log \sin A = 9.5315320 \\ \log \sin 19^\circ\, 52' = 9.5312649 \\ \hline 2671 = \text{diff.} \end{array}$$

$$\text{Tab. diff.} = 3494$$

$$\therefore \frac{2671 \times 60''}{3494} = 46''$$

$$\therefore A = 19^\circ\, 52'\, 46'', \text{ and}$$

$$B = 90^\circ - 19^\circ\, 52'\, 46'' = 70^\circ\, 7'\, 14''.$$

(4.) $$\log \sin A = 10 + \log a - \log c$$

$$\left.\begin{aligned} 10 + \log a = 10 + \log 123 &= 12.0899051 \\ \log c = \log 157 &= 2.1958997 \end{aligned}\right\}\text{subtract}$$

$$\begin{aligned} \therefore \log \sin A &= 9.8940054 \\ \log \sin 51^\circ\, 34' &= 9.8939458 \\ & \quad\quad\;\; 596 = \text{diff.} \end{aligned}$$

$$\text{Tab. diff.} = 1002 \;\therefore \frac{596 \times 60''}{1002} = 35''$$

$$\therefore A = 51^\circ\, 34'\, 35'' \text{ and } B = 90^\circ - 51^\circ\, 34'\, 35'' = 38^\circ\, 25'\, 25''$$

$$2 \log b = \log (c + a) + \log (c - a)$$

$$\begin{aligned} c &= 157 \\ a &= 123 \end{aligned}$$

$$\begin{aligned} \therefore c + a = 280 \text{ and } \log 280 &= 2.4471580 \\ c - a = 34 \text{ and } \log 34 &= 1.5414789 \\ & \;\; 2)3.9786369 \\ \therefore \log b &= 1.9893184 \\ \log 97570 &= .9893163 \\ & \quad\quad\quad\;\; 21 = \text{diff.} \end{aligned}$$

$$\text{Tab. diff.} = 45 \;\therefore \frac{21}{45} = .47$$

$$\therefore b = 97.57047$$

Page 23, *Case* II.

(5.) $$\log \sin A = 10 + \log a - \log c$$

$$\left.\begin{array}{r} 10 + \log a = 10 + \log 576 = 12.7604225 \\ \log c = \log 880 = 2.9444827 \end{array}\right\} \text{subtract}$$

$$\begin{array}{r} \log \sin A = 9.8159398 \\ \log \sin 40^\circ 53' = 9.8159235 \\ \hline 163 = \text{diff.} \end{array}$$

$$\text{Tab. diff.} = 1459 \therefore \frac{163 \times 60''}{1459} = 7''$$

$$\therefore A = 40^\circ 53' 7'' \text{ and}$$

$$B = 90^\circ - 40^\circ 53' 7'' = 49^\circ 6' 53''.$$

(6.) $$\log \sin A = 10 + \log a - \log c$$

$$\left.\begin{array}{r} 10 + \log a = 10 + \log 21.7 = 11.3364597 \\ \log c = \log 54.31 = 1.7348798 \end{array}\right\} \text{subtract}$$

$$\begin{array}{r} \therefore \log \sin A = 9.6015799 \\ \log \sin 23^\circ 33' = 9.6015703 \\ \hline 96 = \text{diff.} \end{array}$$

$$\text{Tab. diff.} = 2897 \therefore \frac{96 \times 60''}{2897} = 2''$$

$$\therefore A = 23\ 33'\ 2'' \text{ and } B = 90^\circ - 23^\circ 33' 2'' = 66^\circ 26' 58''$$

$$2 \log b = \log (c + a) + \log (c - a)$$

$$\begin{array}{r} c = 54.31 \\ a = 21.7 \\ \hline 76.01 \end{array}$$

$$\therefore c+a=76.01 \text{ and } \log 76.01 = 1.8808707$$
$$c-a=32.61 \text{ and } \log 32.61 = 1.5133508$$
$$2)\overline{3.3942215}$$
$$\therefore \log b = 1.6971107$$
$$\log 49786 = .6971072$$
$$35 = \text{diff.}$$

$$\text{Tab. diff.} = 87 \ \therefore \frac{35}{87} = .4$$

$$\therefore b = 49.7864$$

Page 23, *Case* III.

(1.) $$B = 90^\circ - A = 90^\circ - 35^\circ 2' = 54^\circ 58$$

$$\log b = 10 + \log a - \log \tan A$$

$$\left.\begin{aligned} 10 + \log a = 10 + \log 13 &= 11.1139434 \\ \log \tan A = \log \tan 35^\circ 2' &= 9.8457644 \end{aligned}\right\}\text{subtract}$$

$$\therefore \log b = 1.2681790$$
$$\log 18543 = .2681800$$

$$\therefore b = 18.543$$

$$\log c = 10 + \log a - \log \sin A$$

$$\left.\begin{aligned} 10 + \log a &= 11.1139434 \\ \log \sin 35^\circ 2' &= 9.7589519 \end{aligned}\right\}\text{subtract}$$

$$\therefore \log c = 1.3549915$$
$$\log 22646 = .3549915$$

$$\therefore c = 22.646$$

(2.) $A = 90^\circ - B = 90^\circ - 58^\circ\, 3'\, 27'' = 31^\circ\, 56'\, 33''$

$$\log b = 10 + \log a - \log \tan A$$

$$\begin{array}{rr}
10 + \log a = 10 + \log 1157 = & 13.0633334 \\
\log \tan 31^\circ\, 56' = & 9.7946641 \\
\hline
 & 3.2686693 \\
(\text{Tab. diff.} = 2814) \times \dfrac{33''}{60''} = & 1548 \\
\hline
\therefore \log b = & 3.2685145 \\
\log 18557 = & .2685078 \\
\hline
 & 67 = \text{diff.}
\end{array}$$

Tab. diff. = 234

$$\therefore \frac{67}{234} = .29$$

$$\therefore b = 1855.729$$

$$\log c = 10 + \log a - \log \sin A$$

$$\begin{array}{rr}
10 + \log a = & 13.0633334 \\
\log \sin 31^\circ\, 56' = & 9.7234000 \\
(\text{Tab. diff.} = 2026) \times \dfrac{33''}{60''} = & 1114 \\
\hline
\therefore \log c = & 3.3398220 \\
\log 21868 = & .3398091 \\
\hline
 & 129 = \text{diff.}
\end{array}$$

Tab. diff. = 199

$$\frac{129}{199} = .648$$

$$\therefore c = 2186.8648$$

(3.) $A = 90° - B = 90° - 36° = 54°$

$$\log b = 10 + \log a - \log \tan A$$

$$\begin{aligned} 10 + \log a = 10 + \log 825 &= 12.9164539 \\ \log \tan A = \log \tan 54° &= 10.1387390 \\ \hline \therefore \log b &= 2.7777149 \\ \log 59939 &= .7777095 \\ \hline & \qquad 54 = \text{diff.} \end{aligned}$$

Tab. diff. = 73

$$\frac{54}{73} = .7$$

$$\therefore b = 599.397$$

$$\log c = 10 + \log a - \log \sin A$$

$$\begin{aligned} 10 + \log a &= 12.9164539 \\ \log \sin 54° &= 9.9079576 \\ \hline & 3.0084963 \\ \log 10197 &= .0084724 \\ \hline & \qquad 239 = \text{diff.} \end{aligned}$$

Tab. diff. = 426

$$\frac{239}{426} = .56$$

$$\therefore c = 1019.756$$

(4.) $B = 90° - A = 90° - 3° 21' = 86° 39'$

$$\log b = 10 + \log a - \log \tan A$$

$$10+\log a = 10+\log 1426 = 13.1541195$$
$$\log \tan A = \log \tan 3^\circ 21' = \underline{8.7674175}$$
$$\therefore \log b = 4.3867020$$
$$\log 24361 = \underline{.3866951}$$
$$69 = \text{diff.}$$

Tab. diff = 179

$$\frac{69}{179} = .38 \quad \therefore b = 24361.38$$

$$\log c = 10 + \log a - \log \sin A$$

$$10+\log a = 13.1541195$$
$$\log \sin 3^\circ 21' = \underline{8.7666747}$$
$$\therefore \log c = 4.3874448$$
$$\log 24403 = \underline{.3874432}$$
$$16 = \text{diff.}$$

Tab. diff. = 178

$$\frac{16}{178} = .09 \quad \therefore c = 24403.09$$

(5.) $B = 90^\circ - A = 90^\circ - 17^\circ\, 30'\, 30'' = 72^\circ\, 29'\, 30''$

$$\log b = 10 + \log a - \log \tan A$$

$$10+\log a = 10+\log 28.75 = 11.4586378$$
$$\log \tan 17^\circ 30' = \underline{9.4987223}$$
$$1.9599155$$
$$(\text{Tab. diff.} = 4403) \times \frac{30''}{60''} = \underline{2201}$$
$$\therefore \log b = 1.9596954$$
$$\log 91137 = \underline{.9596947}$$
$$7 = \text{diff.}$$

$$\text{Tab. diff.} = 48$$

$$\frac{7}{48} = .1 \therefore b = 91.1371$$

$$\log c = 10 + \log a - \log \sin A$$

$$\begin{array}{rr}
10 + \log a = & 11.4586378 \\
\log \sin 17^\circ 30' = & 9.4781418 \\
\hline
 & 1.9804960 \\
(\text{Tab. diff.} = 4005) \times \dfrac{30''}{60''} = & 2002 \\
\hline
\therefore \log c = & 1.9802958 \\
\log 95564 = & .9802943 \\
\hline
 & 15 = \text{diff.}
\end{array}$$

$$\text{Tab. diff.} = 45$$

$$\frac{15}{45} = .3 \therefore c = 95.5643$$

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(6.) $$B = 90^\circ - A = 90^\circ - 18^\circ = 72^\circ$$

$$\log b = 10 + \log a - \log \tan A$$

$$\begin{array}{rr}
10 + \log a = 10 + \log 1000 = & 13.0000000 \\
\log \tan A = \log \tan 18^\circ = & 9.5117760 \\
\hline
\therefore \log b = & 3.4882240 \\
\log 30776 = & .4882122 \\
\hline
 & 118 = \text{diff.}
\end{array}$$

Tab. diff. = 141

$$\frac{118}{141} = .8368 \ \therefore\ b = 3077.68368$$

$$\log c = 10 + \log a - \log \sin A$$

$$\begin{aligned} 10 + \log a &= 13.0000000 \\ \log \sin 18^\circ &= \underline{9.4899824} \\ \therefore \log e &= 3.5100176 \\ \log 32360 &= \underline{.5100085} \end{aligned}$$

$$91 = \text{diff.}$$

Tab. diff. = 134

$$\frac{91}{134} = .6791 \ \therefore\ c = 3236.06791$$

Case IV.

(1.) $B = 90^\circ - A = 90^\circ - 39^\circ\ 48' = 50^\circ\ 12'$

$$\log a = \log c + \log \sin A - 10$$

$$\left.\begin{aligned} \log c = \log 100 &= 2.0000000 \\ \log \sin A = \log \sin 39^\circ 48' &= \underline{9.8062544} \end{aligned}\right\} \text{Add and reject 10 from the sum.}$$

$$\begin{aligned} \therefore \log a &= 1.8062544 \\ \log 64011 &= \underline{.8062546} \end{aligned}$$

$\therefore a = 64.011$ *very nearly.*

$$\log b = \log c + \log \cos A - 10$$

$$\begin{aligned} \log c = \log 100 &= 2.0000000 \\ \log \cos A = \log \cos 39^\circ 48' &= \underline{9.8855215} \\ \therefore \log b &= 1.8855215 \\ \log 76828 &= \underline{.8855195} \end{aligned}$$

$$20 = \text{diff.}$$

$$\text{Tab. diff.} = 57$$

$$\frac{20}{57} = .35 \therefore b = 76.82835$$

(2.) $A = 90° - B = 90° - 22° \, 3' \, 56'' = 67° \, 56' \, 4''$

$$\log a = \log c + \log \sin A - 10$$

$$\left.\begin{array}{r} \log 1760 = 3.2455127 \\ \log \sin 67° \, 56' = 9.9669614 \\ (\text{Tab. diff.} = 513) \times \frac{4''}{60''} = \quad 34 \end{array}\right\} \begin{array}{l} \text{Add,} \\ \text{(Rule XIII.} \\ \text{App.)} \end{array}$$

$$\begin{array}{r} \log a = 3.2124775 \\ \log 16310 = .2124540 \\ \hline 235 = \text{diff.} \end{array}$$

$$\text{Tab. diff.} = 266$$

$$\frac{235}{266} = .883 \therefore a = 1631.0883$$

$$\log b = \log c + \log \cos A - 10$$

$$\left.\begin{array}{r} \log 1760 = 3.2455127 \\ \log \cos 67° \, 56' = 9.5748240 \end{array}\right\} \text{add}$$

$$\left.\begin{array}{r} 2.8203367 \\ (\text{Tab. diff.} = 3116) \times \frac{4''}{60''} = \quad 208 \end{array}\right\} \begin{array}{l} \text{subtract,} \\ \text{(Rule XIII.)} \end{array}$$

$$\begin{array}{r} \therefore \log b = 2.8203159 \\ \log 66117 = .8203131 \\ \hline 28 = \text{diff.} \end{array}$$

$$\text{Tab. diff.} = 66$$

$$\frac{28}{66} = .424 \therefore b = 661.17424$$

(3.) $B = 90^\circ - A = 90^\circ - 18^\circ\ 5'\ 12'' = 71^\circ\ 54'\ 48''$

$$\log a = \log c + \log \sin A - 10$$

$$\left.\begin{array}{rr} \log 28.347 = & 1.4525071 \\ \log \sin 18^\circ\ 5' = & 9.4919216 \\ (\text{Tab. diff} = 3867) \times \dfrac{12''}{60''} = & 773 \end{array}\right\} \begin{array}{l}\text{add,} \\ \text{(Rule XIII.)}\end{array}$$

$$\begin{array}{rr} \therefore \log a = & .9445060 \\ \log 88004 = & .9445024 \\ \hline & 36 = \text{diff.} \end{array}$$

Tab. diff. = 49

$$\frac{36}{49} = .7347 \quad \therefore a = 8.80047347$$

$$\log b = \log c + \log \cos A - 10$$

$$\left.\begin{array}{rr} \log 28.347 = & 1.4525071 \\ \log \cos 18^\circ\ 5' = & 9.9780006 \end{array}\right\} \text{add}$$

$$\left.\begin{array}{rr} & 1.4305077 \\ (\text{Tab. diff.} = 413) \times \dfrac{12''}{60''} = & 83 \end{array}\right\} \begin{array}{l}\text{subtract,} \\ \text{(Rule XIII.)}\end{array}$$

$$\begin{array}{rr} \therefore \log b = & 1.4304994 \\ \log 26946 = & .4304943 \\ \hline & 51 = \text{diff.} \end{array}$$

Tab. diff. = 161

$$\frac{51}{161} = .32 \textit{ nearly}, \quad \therefore b = 26.94632$$

(4.) $B = 90^\circ - 31^\circ\ 21'\ 6'' = 58^\circ\ 38'\ 54''$

$$\log a = \log c + \log \sin A - 10$$

$$\left.\begin{aligned} \log 897.3 &= 2.9529377 \\ \log \sin 31^\circ 21' &= 9.7162243 \\ (\text{Tab. diff.} = 2073) \times \frac{6''}{60''} &= \qquad\quad 207 \end{aligned}\right\}\text{add}$$

$$\begin{aligned} \therefore \log a &= 2.6691827 \\ \log 46685 &= \ \ .6691774 \\ \hline & \qquad\quad 53 = \text{diff.} \end{aligned}$$

$$\text{Tab. diff.} = 93$$

$$\frac{53}{93} = .57 \ \therefore a = 466.8557$$

$$\log b = \log c + \log \cos A - 10$$

$$\left.\begin{aligned} \log 897.3 &= 2.9529377 \\ \log \cos 31^\circ 21' &= 9.9314605 \end{aligned}\right\}\text{add}$$

$$\left.\begin{aligned} & 2.8843982 \\ (\text{Tab. diff.} = 770) \times \frac{6''}{60''} = & \qquad\quad 77 \end{aligned}\right\}\text{subtract}$$

$$\begin{aligned} \therefore \log b &= 2.8843905 \\ \log 76628 &= \ \ .8843875 \\ \hline & \qquad\quad 30 = \text{diff.} \end{aligned}$$

$$\text{Tab. diff.} = 57$$

$$\frac{30}{57} = .52 \ \therefore b = 766.2852$$

(5.) $$A = 90^\circ - 10^\circ = 80^\circ$$

$$\log a = \log c + \log \sin A - 10$$

$$\left.\begin{aligned} \log 1013 &= 3.0056094 \\ \log \sin 80^\circ &= 9.9933515 \end{aligned}\right\}\text{add}$$

$$\begin{aligned} \therefore \log a &= 2.9989609 \\ \log 99761 &= \ \ .9989608 \end{aligned}$$

$$\therefore a = 997.61$$

$$\log b = \log c + \log \cos A - 10$$

$$\left.\begin{aligned} \log 1013 &= 3.0056094 \\ \log \cos 80^\circ &= 9.2396702 \end{aligned}\right\} \text{add}$$

$$\begin{aligned} \therefore \log b &= 2.2452796 \\ \log 17590 &= .2452658 \end{aligned}$$

$$138 = \text{diff.}$$

Tab. diff. = 246

$$\frac{138}{246} = .5 \therefore b = 175.905$$

(6.) $$B = 90^\circ - 6^\circ\ 13'\ 40'' = 83^\circ\ 46'\ 20''$$

$$\log a = \log c + \log \sin A - 10$$

$$\left.\begin{aligned} \log 50 &= 1.6989700 \\ \log \sin 6^\circ\ 13' &= 9.0345825 \\ (\text{Tab. diff.} = 11582) \times \frac{40''}{60''} &= 7721 \end{aligned}\right\} \text{add}$$

$$\begin{aligned} \therefore \log a &= 0.7343246 \\ \log 54240 &= .7343197 \end{aligned}$$

$$49 = \text{diff.}$$

Tab. diff. = 80

$$\frac{49}{80} = .61 \therefore a = 5.424061$$

$$\log b = \log c + \log \cos A - 10$$

$$\left.\begin{aligned} \log 50 &= 1.6989700 \\ \log \cos 6^\circ\ 13' &= 9.9974386 \end{aligned}\right\} \text{add}$$

$$\left.\begin{aligned} &1.6964086 \\ (\text{Tab. diff.} = 138) \times \frac{40''}{60''} = &92 \end{aligned}\right\} \text{subtract}$$

$$1.6963994$$

$$\therefore \log b = 1.6963994$$
$$\log 49704 = .6963913$$
$$81 = \text{diff.}$$
$$\text{Tab. diff.} = 87$$
$$\frac{81}{87} = .93 \therefore b = 49.70493$$

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(2.) $28.71 \times 103.22 = 2963.4462$

Natural sin $30^\circ = .5$ or $\frac{1}{2}$ (Table II., Manual.)

$\therefore$ Area $= \frac{1}{4} \times 2963.4462 = 740.86155$ sq. ft. *Ans.*

(3.) $123.2 \times 76 = 9363.2$

sin $49^\circ = .75471$ (Manual, Table II.)

$9363.2 \times .75471 = 7066.500672$; half of this is the area.

$\therefore$ Area $= 3533.250336$ sq. ft. *Ans.*

(4.) $b = 1000$ yds. $= \frac{1000}{1760}$ mile $= \frac{25}{44}$ mile

$$c = 2\tfrac{1}{2} = \frac{5}{2} \text{ miles}$$

$\frac{25}{44} \times \frac{5}{2} = \frac{125}{88}$ and sin $42^\circ = .66913$

$\therefore$ Area $= \frac{1}{2} \times \frac{125}{88} \times .66913 = \frac{125 \times .66913}{176}$

$= \frac{83.64125}{176} = .47523$ sq. miles. *Ans.*

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(6. Supplement of $97^\circ = 83^\circ$

$$
\begin{aligned}
\log 513 &= 2.7101174 \\
\log 22 &= 1.3424227 \\
\log \sin 83^\circ &= 9.9967507 \\
\hline
& 14.0492998 \\
& 10. \\
\hline
\therefore \log (2 \text{ area}) &= 4.0492998 \\
\log 11201 &= .0492568 \\
\hline
& 340 = \text{diff.}
\end{aligned}
$$

$$\text{Tab. diff.} = 388$$

$$\frac{340}{388} = .876 \therefore 2 \text{ area} = 11201.876$$

and area $= 5600.938$. *Ans.*

(7.) Supp. $A = 180^\circ - 126^\circ = 54^\circ$

$$
\begin{aligned}
\log 127.3 &= 2.1048284 \\
\log 892.7 &= 2.9507055 \\
\log \sin 54^\circ &= 9.9079576 \\
\hline
& 14.9634915 \\
& 10. \\
\hline
\log (2 \text{ area}) &= 4.9634915 \\
\log 91937 &= .9634903 \\
\hline
& 12 = \text{diff.}
\end{aligned}
$$

$$\text{Tab. diff.} = 47$$

$$\frac{12}{47} = .25 \textit{ nearly}, \therefore 2 \text{ area} = 91937.25$$

and area $= 45968.625$. *Ans.*

(8.)

$$\log 2.314 = 0.3643634$$
$$\log 1.527 = 0.1838390$$
$$\log \sin 49^\circ 6' = 9.8784376$$
$$(\text{Tab. diff.} = 1095) \times \frac{20''}{60''} = \quad 365$$
$$10.4266765$$
$$10.$$
$$\log (2 \text{ area}) = 0.4266765$$
$$\log 26710 = .4266739$$
$$26 = \text{diff.}$$
$$\text{Tab. diff.} = 162$$
$$\frac{26}{162} = .16 \therefore 2 \text{ area} = 2.671016$$

and area $= 1.35508$. *Ans.*

(9.)

$$\log 77 = 1.8864907$$
$$\log 159 = 2.2013971$$
$$\log \sin 50^\circ 31' = 9.8875102$$
$$(\text{Tab. diff.} = 1041) \times \frac{28''}{60''} = \quad 486$$
$$13.9754466$$
$$10.$$
$$\log (2 \text{ area}) = 3.9754466$$
$$\log 94503 = .9754456$$
$$10 = \text{diff.}$$
$$\text{Tab. diff.} = 46$$
$$\frac{10}{46} = .2173 \therefore 2 \text{ area} = 9450.32173$$

and area $= 4725.16086$. *Ans.*

(10.) Supp. $A = 180^\circ - 114^\circ\ 28'\ 32'' = 65^\circ\ 31'\ 28''$

$$\begin{aligned}
\log 287.1 &= 2.4580332 \\
\log 310.25 &= 2.4917118 \\
\log \sin 65^\circ\ 31' &= 9.9590805 \\
(\text{Tab. diff.} = 576) \times \frac{28''}{60''} &= 269 \\
\hline
& 14.9088524 \\
& 10. \\
\hline
\log (2 \text{ area}) &= 4.9088524 \\
\log 81068 &= .9088495 \\
\hline
& 29 = \text{diff.}
\end{aligned}$$

Tab. diff. = 54

$\frac{29}{54} = .537 \therefore$ 2 area = 81068.537

and area = 40534.268. *Ans.*

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(2.)

$$\begin{array}{r}
14.26 \\
19.2 \\
32. \\
\hline
2)65.46 \\
\hline
\end{array}$$

$$\begin{array}{rrr}
32.73 & 32.73 & 32.73 \\
14.26 & 19.2 & 32. \\
\hline
18.47 & 13.53 & .73
\end{array}$$

$\therefore$ Area $= \sqrt{(32.73 \times 18.47 \times 13.53 \times .73)}$

$= \sqrt{(6036.51420639)} = 77.695.$ *Ans.*

(3.)

$$\begin{array}{r} 72. \\ 64.1 \\ 22.3 \\ \hline 2)158.4 \\ \hline 79.2 \end{array}$$

$$\begin{array}{r} 79.2 \\ 72. \\ \hline 7.2 \end{array} \qquad \begin{array}{r} 79.2 \\ 64.1 \\ \hline 15.1 \end{array} \qquad \begin{array}{r} 79.2 \\ 22.3 \\ \hline 56.9 \end{array}$$

$\therefore$ Area $= \sqrt{(79.2 \times 7.2 \times 15.1 \times 56.9)}$

$= \sqrt{(489944.5056)} = 699.96.$ *Ans.*

(4.

$$\begin{array}{r} 53 \\ 49 \\ 98 \\ \hline 2)200 \\ 100 \end{array}$$

$$\begin{array}{r} 100 \\ 53 \\ \hline 47 \end{array} \qquad \begin{array}{r} 100 \\ 49 \\ \hline 51 \end{array} \qquad \begin{array}{r} 100 \\ 98 \\ \hline 2 \end{array}$$

$\therefore$ Area $= \sqrt{(100 \times 47 \times 51 \times 2)} = \sqrt{(479400)}$

$= 692.387.$ *Ans.*

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(6.)

$$\begin{array}{r} 131 \\ 246 \\ 327 \\ \hline 2)704 \\ \hline 352 \end{array}$$

$$\begin{array}{r} 352 \\ 131 \\ \hline 221 \end{array} \qquad \begin{array}{r} 352 \\ 246 \\ \hline 106 \end{array} \qquad \begin{array}{r} 352 \\ 327 \\ \hline 25 \end{array}$$

$$\begin{aligned}
\log 352 &= 2.5465427\\
\log 221 &= 2.3443923\\
\log 106 &= 2.0253059\\
\log 25 &= 1.3979400\\
\hline
&\ 2)8.3141809\\
\hline
\therefore \log \text{area} &= 4.1570904\\
\log 14357 &= \ \ .1570637\\
\hline
&\qquad 267 = \text{diff.}
\end{aligned}$$

Tab. diff. = 302

$\frac{267}{302} = .88 \therefore$ area = 14357.88. *Ans.*

(7.)

$$\begin{array}{r}
2.05\\
1.67\\
2.70\\
\hline
2)6.42\\
\hline
\end{array}$$

$$\begin{array}{rcrcr}
3.21 & & 3.21 & & 3.21\\
2.05 & & 1.67 & & 2.70\\
\hline
1.16 & & 1.54 & & .51
\end{array}$$

$$\begin{aligned}
\log 3.21 &= 0.5065050\\
\log 1.16 &= 0.0644580\\
\log 1.54 &= 0.1875207\\
\log 0.51 &= \bar{1}.7075702\\
\hline
&\ 2)0.4660539\\
\hline
\log \text{area} &= 0.2330269\\
\log 17101 &= \ \ .2330215\\
\hline
&\qquad 54 = \text{diff.}
\end{aligned}$$

Tab. diff. = 254

$\frac{54}{254} = .212 \therefore$ area = 1.7101212. *Ans.*

(8.)

$$\begin{array}{r} 1800 \\ 1728 \\ 1521 \\ \hline 2)5049 \\ \hline 2524.5 \end{array}$$

$$\begin{array}{r} 2524.5 \\ 1800. \\ \hline 724.5 \end{array} \qquad \begin{array}{r} 2524.5 \\ 1728. \\ \hline 796.5 \end{array} \qquad \begin{array}{r} 2524.5 \\ 1521. \\ \hline 1003.5 \end{array}$$

$$\begin{array}{rl} \log 2524.5 = & 3.4021754 \\ \log 724.5 = & 2.8600384 \\ \log 796.5 = & 2.9011858 \\ \log 1003.5 = & 3.0015174 \\ \hline & 2)12.1649170 \\ \hline \log \text{area} = & 6.0824585 \\ \log 12090 = & .0824263 \\ \hline & 322 = \text{diff.} \end{array}$$

$$\text{Tab. diff.} = 359$$

$$\frac{322}{359} = .897 \therefore \text{area} = 1209089.7. \quad \textit{Ans.}$$

N. B.—The area, correct to the second place of decimals, is 1209089.53.

(9.)

$$\begin{array}{r} 0.23 \\ 0.34 \\ 0.45 \\ \hline 2)1.02 \\ \hline 0.51 \end{array}$$

$$\begin{array}{r} 0.51 \\ .23 \\ \hline .28 \end{array} \qquad \begin{array}{r} 0.51 \\ .34 \\ \hline .17 \end{array} \qquad \begin{array}{r} 0.51 \\ .45 \\ \hline .06 \end{array}$$

$$\left.\begin{array}{l}\log .51 = \bar{1}.7075702 \\ \log .28 = \bar{1}.4471580 \\ \log .17 = \bar{1}.2304489 \\ \log .06 = \bar{2}.7781513\end{array}\right\}\text{(Rule II. App.)}$$

$$2)\bar{3}.1633284$$

$$\log \text{area} = \bar{2}.5816642 \quad \text{(See App. p. 63, N. B.)}$$
$$\log 38164 = .5816539$$
$$103$$

Tab. diff. = 114

$$\frac{103}{114} = .9 \therefore \text{area} = .0381649. \quad \textit{Ans.}$$

(10.)

$$\begin{array}{r} 507 \\ 603 \\ 721 \\ \hline 2)1831 \\ \hline 915.5 \end{array}$$

$$\begin{array}{r} 915.5 \\ 507. \\ \hline 408.5 \end{array} \qquad \begin{array}{r} 915.5 \\ 603. \\ \hline 312.5 \end{array} \qquad \begin{array}{r} 915.5 \\ 721. \\ \hline 194.5 \end{array}$$

$$\begin{array}{l} \log 915.5 = 2.9616583 \\ \log 408.5 = 2.6111921 \\ \log 312.5 = 2.4948500 \\ \log 194.5 = 2.2889196 \end{array}$$

$$2)10.3566200$$

$$\log \text{area} = 5.1783100$$
$$\log 15076 = .1782861$$
$$239 = \text{diff.}$$

Tab. diff. = 288

$$\frac{239}{288} = .83 \therefore \text{area} = 150768.3. \quad \textit{Ans.}$$

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(2.) $B + C = 43^\circ\ 31' + 71^\circ\ 25' = 114^\circ\ 56'$

$\therefore A = 180^\circ - 114^\circ\ 56' = 65^\circ\ 4'$

$\log b = \log a + \log \sin B - \log \sin A$

$$\left.\begin{array}{r} \log 14.83 = 1.1711412 \\ \log \sin 43^\circ\ 31' = 9.8379453 \end{array}\right\}\text{add}$$

$$\left.\begin{array}{r} 11.0090865 \\ \log \sin 65^\circ\ 4' = 9.9575110 \end{array}\right\}\text{subtract}$$

$$\begin{array}{r} \therefore \log b = 1.0515755 \\ \log 11260 = .0515384 \\ \hline 371 = \text{diff.} \end{array}$$

Tab diff. = 386

$$\frac{371}{386} = .96 \therefore b = 11.26096$$

$\log c = \log a + \log \sin C - \log \sin A$

$$\left.\begin{array}{r} \log 14.83 = 1.1711412 \\ \log \sin 71^\circ\ 25' = 9.9767447 \end{array}\right\}\text{add}$$

$$\left.\begin{array}{r} 11.1478859 \\ \log \sin 65^\circ\ 4' = 9.9575110 \end{array}\right\}\text{subtract}$$

$$\begin{array}{r} \therefore \log c = 1.1903749 \\ \log 15501 = .1903597 \\ \hline 152 = \text{diff.} \end{array}$$

$$\text{Tab. diff.} = 280$$

$$\frac{152}{280} = .54 \therefore c = 15.50154$$

(3.) $B + C = 72^\circ\, 31'\, 30'' + 81^\circ\, 24'\, 20''$

$= 153^\circ\, 55'\, 50''$

$$\therefore A = 180^\circ - 153^\circ\, 55'\, 50'' = 26^\circ\, 4'\, 10''$$

$$\log b = \log a + \log \sin B - \log \sin A$$

$$\left.\begin{array}{r} \log 1728 = 3.2375437 \\ \log \sin 72^\circ\, 31' = 9.9794593 \\ (\text{Tab. diff.} = 398) \times \dfrac{30''}{60''} = \qquad 199 \end{array}\right\} \text{add}$$

$$\left.\begin{array}{r} 13.2170229 \\ \log \sin 26^\circ\, 4' = 9.6428765 \end{array}\right\} \text{subtract}$$

$$\left.\begin{array}{r} 3.5741464 \\ (\text{Tab. diff.} = 2582) \times \dfrac{10''}{60''} = \qquad 430 \end{array}\right\} \text{subtract}$$

$$\therefore \log b = 3.5741034$$

$$\log 37506 = .5741007$$

$$27 = \text{diff.}$$

$$\text{Tab. diff.} = 115$$

$$\frac{27}{115} = .23 \therefore b = 3750.623$$

$$\log c = \log a + \log \sin C - \log \sin A$$

$$\left.\begin{array}{r} \log 1728 = 3.2375437 \\ \log \sin 81^\circ\, 24' = 9.9950893 \\ (\text{Tab. diff.} = 191) \times \dfrac{20''}{60''} = \qquad 64 \end{array}\right\} \text{add}$$

$$13.2326394$$

$$\left.\begin{array}{rr} & 13.2326394 \\ \log \sin 26^\circ 4' = & 9.6428765 \end{array}\right\}\text{subtract}$$

$$\left.\begin{array}{rr} & 3.5897629 \\ (\text{Tab. diff.} = 2582) \times \dfrac{10''}{60''} = & 430 \end{array}\right\}\text{subtract}$$

$$\begin{array}{rr} \therefore \log c = & 3.5897199 \\ \log 38879 = & .5897151 \\ \hline & 48 = \text{diff.} \end{array}$$

$$\text{Tab. diff.} = 111$$

$$\frac{48}{111} = .43 \quad \therefore c = 3887.943$$

(4.) $B + C = 117^\circ 23' 12'' + 52^\circ 18' 10''$
$= 169^\circ 41' 22''$

$$\therefore A = 180^\circ - 169^\circ 41' 22'' = 10^\circ 18' 38''$$

$$\text{Supp. } B = 180^\circ - 117^\circ 23' 12'' = 62^\circ 36' 48''$$

$$\log b = \log a + \log \sin B - \log \sin A$$

$$\left.\begin{array}{rr} \log 537.21 = & 2.7301441 \\ \log \sin 62^\circ 36' = & 9.9483227 \\ (\text{Tab. diff.} = 655) \times \dfrac{48''}{60''} = & 524 \end{array}\right\}\text{add}$$

$$\left.\begin{array}{rr} & 12.6785192 \\ \log \sin 10^\circ 18' = & 9.2523729 \end{array}\right\}\text{subtract}$$

$$\left.\begin{array}{rr} & 3.4261463 \\ (\text{Tab. diff.} = 6946) \times \dfrac{38''}{60''} = & 4399 \end{array}\right\}\text{subtract}$$

$$\begin{array}{rr} \therefore \log b = & 3.4257064 \\ \log 26650 = & .4256972 \\ \hline & 92 = \text{diff.} \end{array}$$

$$\text{Tab. diff.} = 163$$

$$\frac{92}{163} = .56 \therefore b = 2665.056$$

$$\log c = \log a + \log \sin C - \log \sin A$$

$$\left.\begin{array}{rr} \log 537.21 = & 2.7301441 \\ \log \sin 52^\circ\ 18' = & 9.8982992 \\ (\text{Tab. diff.} = 977) \times \frac{10''}{60''} = & 163 \end{array}\right\}\text{add}$$

$$\left.\begin{array}{rr} & 12.6284596 \\ \log \sin 10^\circ\ 18' = & 9.2523729 \end{array}\right\}\text{subtract}$$

$$3.3760867$$

$$\left.\begin{array}{rr} (\text{Tab. diff.} = 6946) \times \frac{38''}{60''} = & 4399 \end{array}\right\}\text{subtract}$$

$$\begin{array}{rr} \therefore \log c = & 3.3756468 \\ \log 23749 = & .3756453 \\ \hline & 15 = \text{diff.} \end{array}$$

$$\text{Tab. diff.} = 183$$

$$\frac{15}{183} = .08 \therefore c = 2374.908$$

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(5.) $B + C = 120^\circ\ 15'\ 15'' + 36^\circ\ 52' = 157^\circ\ 7'\ 15''$

$$\therefore A = 180^\circ - 157^\circ\ 7'\ 15'' = 22^\circ\ 52'\ 45''$$

$$\text{Supp. } B = 180^\circ - 120^\circ\ 15'\ 15'' = 59^\circ\ 44'\ 45''$$

$$\log b = \log a + \log \sin B - \log \sin A$$

$$\left.\begin{array}{r} \log 1000 = 3.0000000 \\ \log \sin 59^\circ 44' = 9.9363574 \\ (\text{Tab. diff.} = 738) \times \frac{45''}{60''} = \qquad 578 \end{array}\right\} \text{add}$$

$$\left.\begin{array}{r} 12.9364152 \\ \log \sin 22^\circ 52' = 9.5894893 \end{array}\right\} \text{subtract}$$

$$\left.\begin{array}{r} 3.3469259 \\ (\text{Tab. diff.} = 2995) \times \frac{45''}{60''} = \qquad 2246 \end{array}\right\} \text{subtract}$$

$$\therefore \log b = 3.3467013$$
$$\log 22217 = .3466854$$

$$\text{Tab. diff.} = 196 \qquad 159 = \text{diff.}$$

$$\frac{159}{196} = .811 \therefore b = 2221.7811$$

$$\log c = \log a + \log \sin C - \log \sin A$$

$$\left.\begin{array}{r} \log 1000 = 3.0000000 \\ \log \sin 36^\circ 52' = 9.7781186 \end{array}\right\} \text{add}$$

$$\left.\begin{array}{r} 12.7781186 \\ \log \sin 22^\circ 52' = 9.5894893 \end{array}\right\} \text{subtract}$$

$$\left.\begin{array}{r} 3.1886293 \\ (\text{Tab. diff.} = 2995) \times \frac{45''}{60''} = \qquad 2246 \end{array}\right\} \text{subtract}$$

$$\therefore \log c = 3.1884047$$
$$\log 15431 = .1883941$$

$$\text{Tab. diff.} = 281 \qquad 106 = \text{diff.}$$

$$\frac{106}{281} = .37 \therefore c = 1543.137$$

(6.) $B+C=36^\circ\ 43'\ 20''+22^\circ\ 10'\ 15''=58^\circ\ 53'\ 35''$

$$\therefore A=180^\circ-58^\circ\ 53'\ 35''=121^\circ\ 6'\ 25''$$

$$\text{and Supp. } A=58^\circ\ 53'\ 35''$$

$$\log b=\log a+\log\sin B-\log\sin A$$

$$\left.\begin{array}{rr} \log 97.6 = & 1.9894498 \\ \log\sin 36^\circ\ 43' = & 9.7765983 \\ (\text{Tab. diff.}=1693)\times\frac{20''}{60''} = & 564 \end{array}\right\}\text{add}$$

$$\left.\begin{array}{rr} & 11.7661045 \\ \log\sin 58^\circ\ 53' = & 9.9325330 \end{array}\right\}\text{subtract}$$

$$\left.\begin{array}{rr} & 1.8335715 \\ (\text{Tab. diff.}=763)\times\frac{35''}{60''} = & 445 \end{array}\right\}\text{subtract}$$

$$\begin{array}{rr} \therefore \log b = & 1.8335270 \\ \log 68159 = & .8335232 \\ & 38=\text{diff.} \end{array}$$

$$\text{Tab. diff.}=64$$

$$\frac{38}{64}=.6\ \therefore\ b=68.1596$$

$$\log c=\log a+\log\sin C-\log\sin A$$

$$\left.\begin{array}{rr} \log 97.6 = & 1.9894498 \\ \log\sin 22^\circ\ 10' = & 9.5766892 \\ (\text{Tab. diff.}=3099)\times\frac{15''}{60''} = & 775 \end{array}\right\}\text{add}$$

$$\left.\begin{array}{rr} & 11.5662165 \\ \log\sin 58^\circ\ 53' = & 9.9325330 \end{array}\right\}\text{subtract}$$

$$\begin{array}{rr} & 1.6336835 \end{array}$$

$$\left.\begin{array}{rr} & 1.63368835 \\ (\text{Tab. diff.}=763)\times\frac{35''}{60''}= & 445 \end{array}\right\}\text{subtract}$$

$$\begin{aligned}\log c &= 1.6336390\\ \log 43016 &= \ .6336300\end{aligned}$$

$$90=\text{diff.}$$

$$\text{Tab. diff.}=100$$

$$\frac{90}{100}=.9 \therefore c=43.0169$$

Page 47.

(3.) By equation (1), page 44, we have,

$$\log \sin B=\log \sin A+\log b-\log a$$

$$\left.\begin{array}{rr} \log \sin 36^\circ\ 42'= & 9.7764289 \\ (\text{Tab. diff.}=1694)\times\frac{30''}{60''}= & 847 \\ \log 53= & 1.7242759 \end{array}\right\}\text{add}$$

$$\left.\begin{array}{rr} & 11.5007895 \\ \log 47= & 1.6720979 \end{array}\right\}\text{subtract}$$

$$\begin{aligned}\therefore \log \sin B &= 9.8286916\\ \log \sin 42^\circ\ 22' &= 9.8285778\end{aligned}$$

$$1138=\text{diff.}$$

$$\text{Tab. diff.}=1385$$

$$\frac{1138\times 60''}{1385}=49'' \therefore B=42^\circ\ 22'\ 49''. \quad (\text{Rule XIV. App.})$$

$$A+B=36^\circ\ 42'\ 30''+42^\circ\ 22'\ 49''=79^\circ\ 5'\ 19''$$

$$\therefore C=180^\circ-79^\circ\ 5'\ 19''=100^\circ\ 54'\ 41''$$

and $\therefore$ Supp. $C = 79° \ 5' \ 19''$

By equation (3), page 42, we have

$$\log c = \log a + \log \sin C - \log \sin A$$

$$\left.\begin{array}{r} \log 47 = 1.6720979 \\ \log \sin 79° \ 5' = 9.9920689 \\ (\text{Tab. diff.} = 244) \times \dfrac{19''}{60''} = 77 \end{array}\right\} \text{add}$$

$$\left.\begin{array}{r} 11.6641745 \\ \log \sin 36° \ 42' = 9.7764289 \end{array}\right\} \text{subtract}$$

$$\left.\begin{array}{r} 1.8877456 \\ (\text{Tab. diff.} = 1694) \times \dfrac{30''}{60''} = 847 \end{array}\right\} \text{subtract}$$

$$\begin{array}{r} \therefore \log c = 1.8876609 \\ \log 77207 = \ .8876567 \\ \hline 42 = \text{diff.} \end{array}$$

Tab. diff. $= 56$

$$\frac{42}{56} = .75 \ \therefore c = 77.20775$$

(4.) Supp. $A = 180° - 124° \ 32' = 55° \ 28'$

$$\log \sin B = \log \sin A + \log b - \log a$$

$$\left.\begin{array}{r} \log \sin 55° \ 28' = 9.9158200 \\ \log 312 = 2.4941546 \end{array}\right\} \text{add}$$

$$\left.\begin{array}{r} 12.4099746 \\ \log 517 = 2.7134905 \end{array}\right\} \text{subtract}$$

$$\begin{array}{r} \therefore \log \sin B = 9.6964841 \\ \log \sin 29° \ 48' = 9.6963336 \\ \hline 1505 = \text{diff.} \end{array}$$

$$\text{Tab. diff.} = 2205$$

$$\frac{1505 \times 60''}{2205} = 41'' \therefore B = 29^\circ\ 48'\ 41''$$

$$A + B = 124^\circ\ 32' + 29^\circ\ 48'\ 41'' = 154^\circ\ 20'\ 41''$$

$$\therefore C = 180^\circ - 154^\circ\ 20'\ 41'' = 25^\circ\ 39'\ 19''$$

$$\log c = \log a + \log \sin C - \log \sin A$$

$$\left.\begin{array}{r} \log 517 = 2.7134905 \\ \log \sin 25^\circ\ 39' = 9.6363601 \\ (\text{Tab. diff.} = 2630) \times \dfrac{19''}{60''} = 833 \end{array}\right\}\text{add}$$

$$\left.\begin{array}{r} 12.3499339 \\ \log \sin 55^\circ\ 28' = 9.9158200 \end{array}\right\}\text{subtract}$$

$$\begin{array}{r} \therefore \log c = 2.4341139 \\ \log 27171 = .4341056 \\ \hline 83 = \text{diff.} \end{array}$$

$$\text{Tab. diff.} = 160$$

$$\frac{83}{160} = .519 \therefore c = 271.71519$$

(5.) $\log \sin B = \log \sin A + \log b - \log a$

$$\left.\begin{array}{r} \log \sin 62^\circ\ 24' = 9.9475335 \\ (\text{Tab. diff.} = 661) \times \dfrac{20''}{60''} = 220 \\ \log 217 = 2.3364597 \end{array}\right\}\text{add}$$

$$\left.\begin{array}{r} 12.2840152 \\ \log 199 = 2.2988531 \end{array}\right\}\text{subtract}$$

$$\begin{array}{r} \therefore \log \sin B = 9.9851621 \\ \log \sin 75^\circ\ 6' = 9.9851462 \\ \hline 159 = \text{diff.} \end{array}$$

$$\text{Tab. diff.} = 337$$

$$\frac{159 \times 60''}{337} = 28'' \therefore \text{ since } B \text{ is } \textit{obtuse},$$

$$B = 180^\circ - 75^\circ\ 6'\ 28'' = 104^\circ\ 53'\ 32''$$

$$A + B = 62^\circ\ 24'\ 20'' + 104^\circ\ 53'\ 32'' = 167^\circ\ 17'\ 52''$$

$$\therefore C = 180^\circ - 167^\circ\ 17'\ 52'' = 12^\circ\ 42'\ 8''$$

$$\log c = \log a + \log \sin C - \log \sin A$$

$$\left.\begin{array}{r} \log 199 = 2.2988531 \\ \log \sin 12^\circ\ 42' = 9.3421190 \\ (\text{Tab. diff.} = 5602) \times \dfrac{8''}{60''} = \qquad 747 \end{array}\right\} \text{add}$$

$$\left.\begin{array}{r} 11.6410468 \\ \log \sin 62^\circ\ 24' = 9.9475335 \end{array}\right\} \text{subtract}$$

$$\left.\begin{array}{r} 1.6935133 \\ (\text{Tab. diff.} = 661) \times \dfrac{20''}{60''} = \qquad 220 \end{array}\right\} \text{subtract}$$

$$\begin{array}{r} \therefore \log c = 1.6934913 \\ \log 49373 = .6934895 \\ \hline 18 = \text{diff.} \end{array}$$

$$\text{Tab. diff.} = 88$$

$$\frac{18}{88} = .204 \therefore c = 49{\cdot}373204$$

(6.) Supp. $A = 180^\circ - 107^\circ\ 3'\ 13'' = 72^\circ\ 56'\ 47''$

$$\log \sin B = \log \sin A + \log b - \log a$$

$$\left.\begin{array}{r} \log \sin 72^\circ\ 56' = 9.9804415 \\ (\text{Tab diff.} = 388) \times \dfrac{47''}{60''} = \qquad 304 \\ \log 30.8 = 1.4885507 \end{array}\right\}\text{add}$$

$$\left.\begin{array}{r} 11.4690226 \\ \log 62.73 = 1.7974753 \end{array}\right\}\text{subtract}$$

$$\begin{array}{r} \therefore \log \sin B = 9.6715473 \\ \log \sin 27^\circ\ 59' = 9.6713716 \\ \hline 1757 = \text{diff.} \end{array}$$

$$\text{Tab. diff.} = 2377$$

$$\frac{1757 \times 60''}{2377} = 44'' \therefore B = 27^\circ\ 59'\ 44''$$

$$A + B = 107^\circ\ 3'\ 13'' + 27^\circ\ 59'\ 44'' = 135^\circ\ 2'\ 57''$$

$$\therefore C = 180^\circ - 135^\circ\ 2'\ 57'' = 44^\circ\ 57'\ 3''$$

$$\log c = \log a + \log \sin C - \log \sin A$$

$$\left.\begin{array}{r} \log 62.73 = 1.7974753 \\ \log \sin 44^\circ\ 57' = 9.8491057 \\ (\text{Tab. diff.} = 1265) \times \dfrac{3''}{60''} = \qquad 63 \end{array}\right\}\text{add}$$

$$\left.\begin{array}{r} 11.6465873 \\ \log \sin 72^\circ\ 56' = 9.9804415 \end{array}\right\}\text{subtract}$$

$$\left.\begin{array}{r} 1.6661458 \\ (\text{Tab. diff.} = 388) \times \dfrac{47''}{60''} = \qquad 304 \end{array}\right\}\text{subtract}$$

$$\begin{array}{r} \therefore \log c = 1.6661154 \\ \log 46357 = \ .6661153 \end{array}$$

$$\therefore c = 46.357$$

Page 48.

(2.) $$a+b=516+219=735$$

$$a-b=516-219=297$$

$$\tfrac{1}{2}(A+B)=90^\circ-\tfrac{1}{2}C=90^\circ-49^\circ\,27'=40^\circ\,33'$$

$$\log\tan\tfrac{1}{2}(A-B)=\log(a-b)+\log\tan\tfrac{1}{2}(A+B)-\log(a+b)$$

$$\left.\begin{array}{r}\log 297=2.4727564\\ \log\tan 40^\circ\,33'=9.9322662\end{array}\right\}\text{add}$$

$$\left.\begin{array}{r}12.4050226\\ \log 735=2.8662873\end{array}\right\}\text{subtract}$$

$$\log\tan\tfrac{1}{2}(A-B)=9.5387353$$

$$\log\tan 19^\circ\,4'=9.5386110$$

$$1243=\text{diff.}$$

Tab. diff. $=4090$

$$\frac{1243\times 60''}{4090}=18''\;\therefore\;\tfrac{1}{2}(A-B)=19^\circ\,4'\,18''$$

$$\text{and }\tfrac{1}{2}(A+B)=40.33$$

$$\therefore A=59^\circ\,37'\,18''$$
$$\text{and }B=21\;\;28\;\;42$$

$$\log c=\log a+\log\sin C-\log\sin A$$

by equation (3), page 42;

and supp. $C=180^\circ-98^\circ\,54'=81^\circ\,6'$

$$\left.\begin{array}{r} \log 516 = 2.7126497 \\ \log \sin 81^\circ\ 6' = 9.9947393 \end{array}\right\}\text{add}$$

$$\left.\begin{array}{r} 12.7073890 \\ \log \sin 59^\circ\ 37' = 9.9358401 \end{array}\right\}\text{subtract}$$

$$\left.\begin{array}{r} 2.7715489 \\ (\text{Tab. diff.} = 741) \times \frac{18''}{60''} = \quad 222 \end{array}\right\}\text{subtract}$$

$$\therefore \log c = 2.7715267$$
$$\log 59091 = .7715213$$
$$54 = \text{diff.}$$

$$\text{Tab. diff.} = 73$$

$$\frac{54}{73} = .74 \ \therefore\ c = 590.9174$$

Page 49.

(3.) $a + b = 53.24 + 31.27 = 84.51$

$a - b = 53.24 - 31.27 = 21.97$

$\frac{1}{2}(A + B) = 90^\circ - \frac{1}{2}C = 90^\circ - 63^\circ\ 18'\ 3'' = 26^\circ\ 41'\ 57''$

$$\log \tan \tfrac{1}{2}(A - B) = \log(a - b) + \log \tan \tfrac{1}{2}(A + B) - \log(a + b)$$

$$\left.\begin{array}{r} \log 21.97 = 1.3418301 \\ \log \tan 26^\circ\ 41' = 9.7012080 \\ (\text{Tab. diff.} = 3147) \times \frac{57''}{60''} = \quad 2990 \end{array}\right\}\text{add}$$

$$11.0433371$$

$$\left.\begin{array}{r} 11.0433371 \\ \log 84.51 = 1.9269081 \end{array}\right\}\text{subtract}$$

$$\therefore \log \tan \tfrac{1}{2}(A-B) = 9.1164290$$
$$\log \tan 7^\circ\, 26' = 9.1155072$$

$$9218 = \text{diff.}$$

Tab. diff. $= 9837$

$$\frac{9218 \times 60''}{9837} = 56'' \therefore \tfrac{1}{2}(A-B) = 7^\circ\, 26'\, 56''$$
$$\tfrac{1}{2}(A+B) = 26^\circ\, 41'\, 57''$$

$$\therefore A = 34^\circ\ \ 8'\, 53''$$
$$B = 19^\circ\, 15'\ \ 1''$$

Supp. $C = 180^\circ - 126^\circ\, 36'\, 6'' = 53^\circ\, 23'\, 54''$

$$\log c = \log a + \log \sin C - \log \sin A$$

$$\left.\begin{array}{r} \log 53.24 = 1.7262380 \\ \log \sin 53^\circ\, 23' = 9.9045230 \\ (\text{Tab. diff} = 939) \times \dfrac{54''}{60''} = \qquad 845 \end{array}\right\}\text{add}$$

$$\left.\begin{array}{r} 11.6308455 \\ \log \sin 34^\circ\, 8' = 9.7490562 \end{array}\right\}\text{subtract}$$

$$\left.\begin{array}{r} 1.8817893 \\ (\text{Tab. diff.} = 1863) \times \dfrac{53''}{60''} = \qquad 1645 \end{array}\right\}\text{subtract}$$

$$\therefore \log c = 1.8816248$$
$$\log 76142 = 1.8816243$$

$$\therefore c = 76.142$$

D

(4) $$a+b=831+536=1367$$
$$a-b=831-536=295$$
$$\tfrac{1}{2}=(A+B)=90^\circ\tfrac{1}{2}C=90-8^\circ\ 14'\ 20''=81^\circ\ 45'\ 40''$$
$$\log\tan\tfrac{1}{2}(A-B)=\log(a-b)+\log\tan\tfrac{1}{2}(A+B)-\log(a+b)$$

$$\left.\begin{array}{rr} \log 295 = & 2.4698220 \\ \log\tan 81^\circ\ 45' = & 10.8386527 \\ (\text{Tab. diff.}=8888)\times\dfrac{40''}{60''}= & 5925 \end{array}\right\}\text{add}$$

$$\left.\begin{array}{rr} & 13.3090672 \\ \log 1367 = & 3.1357685 \end{array}\right\}\text{subtract}$$

$$\therefore \log\tan\tfrac{1}{2}(A-B)=10.1732987$$
$$\log\tan 56^\circ\ 8'=10.1731947$$
$$1040=\text{diff.}$$

Tab. diff. = 2730

$$\frac{1040\times 60''}{2730}=23''\quad\therefore\ \tfrac{1}{2}(A-B)=56^\circ\ \ 8'\ 23''$$
$$\tfrac{1}{2}(A+B)=81^\circ\ 45'\ 40''$$
$$\therefore A=137^\circ\ 54'\ 3''$$
$$\text{and } B=\ 25\ \ 37\ \ 17$$

$$\text{Supp. } A=180^\circ-137^\circ\ 54'\ 3''=42^\circ\ 5'\ 57''$$
$$\log c=\log a+\log\sin C-\log\sin A$$

$$\left.\begin{array}{rr} \log 831 = & 2.9196010 \\ \log\sin 16^\circ\ 28' = & 9.4524879 \\ (\text{Tab. diff.}=4272)\times\dfrac{40''}{60''}= & 2848 \end{array}\right\}\text{add}$$

$$12.3723737$$

$$\begin{array}{rr} & 12.3723737 \\ \log \sin 42^\circ\ 5' = & 9.8262114 \end{array} \Big\} \text{subtract}$$

$$\begin{array}{rr} & 2.5461623 \\ (\text{Tab. diff.} = 1398) \times \dfrac{57''}{60''} = & 1328 \end{array} \Big\} \text{subtract}$$

$$\begin{array}{rr} \therefore \log c = & 2.5460295 \\ \log 35158 = & .5460242 \\ \hline & 53 = \text{diff.} \end{array}$$

$$\text{Tab. diff.} = 124$$

$$\frac{53}{124} = .4 \quad \therefore c = 351.584$$

(5.) $a + b = 8214 + 3732 = 11946$

$a - b = 8214 - 3732 = 4482$

$\frac{1}{2}(A + B) = 90^\circ - \frac{1}{2}C = 90^\circ - 30^\circ\ 56'\ 30'' = 59^\circ\ 3'\ 30''$

$$\log \tan \tfrac{1}{2}(A - B) = \log (a - b) + \log \tan \tfrac{1}{2}(A + B) - \log (a + b)$$

$$\left.\begin{array}{rr} \log 4482 = & 3.6514719 \\ \log \tan 59^\circ\ 3' = & 10.2220851 \\ (\text{Tab. diff.} = 2863) \times \dfrac{30''}{60''} = & 1431 \end{array}\right\} \text{add}$$

$$\left.\begin{array}{rr} & 13.8737001 \\ \log 11946 = & 4.0772225 \end{array}\right\} \text{subtract}$$

$$\begin{array}{rr} \log \tan \tfrac{1}{2}(A - B) = & 9.7964776 \\ \log \tan 32^\circ\ 2' = & 9.7963531 \\ \hline & 1263 = \text{diff.} \end{array}$$

$$\text{Tab. diff.} = 2809$$

$$\frac{1263 \times 60''}{2809} = 27'' \therefore \tfrac{1}{2}(A - B) = 32^\circ\ 2'\ 27''$$

$$\tfrac{1}{2}(A + B) = 59\ \ 3\ \ 30$$

$$\therefore A = 91^\circ\ 5'\ 57''$$

$$\text{and } B = 27\ \ 1\ \ \ 3$$

$$\text{Supp. } A = 180^\circ - 91^\circ\ 5'\ 57' = 88^\circ\ 54'\ 3''$$

$$\log c = \log a + \log \sin C - \log \sin A$$

$$\left.\begin{aligned} \log 8214 &= 3.9145547 \\ \log \sin 61^\circ\ 53' &= 9.9454636 \end{aligned}\right\}\text{add}$$

$$\left.\begin{aligned} &13.8600183 \\ \log \sin 88^\circ\ 54' &= 9.9999200 \end{aligned}\right\}\text{subtract}$$

$$\left.\begin{aligned} &3.8600983 \\ (\text{Tab. diff.} = 25) \times \frac{3''}{60''} &= \qquad\qquad 1 \end{aligned}\right\}\text{subtract}$$

$$\therefore \log c = 3.8600982$$

$$\log 72460 = 3.8600983$$

$$\therefore c = 7246$$

(6.) $$a + b = 1.73 + 1.23 = 2.96$$

$$a - b = 1.73 - 1.23 = 0.5$$

$$\tfrac{1}{2}(A + B) = 90^\circ - \tfrac{1}{2}C = 90^\circ - 11^\circ\ 6'\ 45 = 78^\circ\ 53'\ 15''$$

$$\log \tan \tfrac{1}{2}(A - B) = \log (a - b) + \log \tan \tfrac{1}{2}(A + B) - \log (a + b)$$

$$\left.\begin{aligned} \log .5 &= \bar{1}.6989700 \\ \log \tan 78^\circ\ 53' &= 10.7066500 \\ (\text{Tab. diff.} = 6672) \times \frac{15''}{60''} &= \qquad\quad 1668 \end{aligned}\right\}\text{add}$$

$$10.4057868$$

$$\left.\begin{array}{r} 10.4057868 \\ \log 2.96 = 0.4712917 \end{array}\right\} \text{subtract}$$

$$\log \tan \tfrac{1}{2}(A-B) = 9.9344951$$
$$\log \tan 40^\circ\ 41' = 9.9343114$$

$$1837 = \text{diff.}$$

Tab. diff. $= 2556$

$$\frac{1837 \times 60''}{2556} = 43'' \therefore \tfrac{1}{2}(A-B) = 40^\circ\ 41'\ 43''$$
$$\tfrac{1}{2}(A+B) = 78\ \ 53\ \ 15$$

$$\therefore A = 119^\circ\ 34'\ 58''$$
$$\text{and } B = 38\ \ 11\ \ 32$$

Supp. $A = 180^\circ - 119^\circ\ 34'\ 58'' = 60^\circ\ 25'\ 2''$

$$\log c = \log a + \log \sin C - \log \sin A$$

$$\left.\begin{array}{r} \log 1.73 = 0.2380461 \\ \log \sin 22^\circ\ 13' = 9.5776183 \\ (\text{Tab. diff.} = 3092) \times \dfrac{30''}{60''} = \qquad 1546 \end{array}\right\} \text{add}$$

$$\left.\begin{array}{r} 9.8158190 \\ \log \sin 60^\circ\ 25' = 9.9393388 \end{array}\right\} \text{subtract}$$

$$\left.\begin{array}{r} \bar{1}.8764802 \\ (\text{Tab. diff.} = 717) \times \dfrac{2''}{60''} = \qquad 24 \end{array}\right\} \text{subtract}$$

$$\therefore \log c = \bar{1}.8764778$$
$$\log 75245 = 0.8764776$$

$$\therefore c = .75245$$

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(2.)

$$\begin{array}{r} a = 15.32 \\ b = 21.56 \\ c = 16.22 \\ \hline 2)53.10 \\ \hline s = 26.55 \\ 21.56 \\ \hline s-b = 4.99 \end{array} \qquad \begin{array}{r} 26.55 \\ 16.22 \\ \hline s-c = 10.33 \end{array}$$

$$\log\sin\tfrac{1}{2}A = 10 + \tfrac{1}{2}\{\log(s-b) + \log(s-c) - \log b - \log c\}$$

$$\begin{array}{r} \log 4.99 = 0.6981005 \\ \log 10.33 = 1.0141003 \\ \hline 1.7122008 \\ 2.5436996 \\ \hline 2)\bar{1}.1685012 \\ \hline \bar{1}.5842506 \\ 10. \\ \hline \end{array}\left.\right\}\text{subtract} \qquad \begin{array}{r} \log 21.56 = 1.3336488 \\ \log 16.22 = 1.2100508 \\ \hline 2.5436996 \end{array}$$

$$\begin{array}{r} \therefore \log\sin\tfrac{1}{2}A = 9.5842506 \\ \log\sin 22^\circ 34' = 9.5840576 \\ \hline 1930 = \text{diff.} \end{array}$$

Tab. diff. = 3039

$$\frac{1930 \times 60''}{3039} = 38'' \quad \therefore \tfrac{1}{2}A = 22^\circ\ 34'\ 38''$$

and $A = 45^\circ\ 9'\ 16''$ *Ans.*

(3.)

$$\begin{array}{r} a = 2134 \\ b = 1617 \\ c = 815 \\ \hline 2)4566 \\ \hline s = 2283 \\ 2134 \\ \hline s - a = 149 \end{array} \qquad \begin{array}{r} 2283 \\ 815 \\ \hline s - c = 1468 \end{array}$$

$$\log \sin \tfrac{1}{2} B = 10 + \tfrac{1}{2} \{ \log (s - a) + \log (s - c) - \log a - \log c \}$$

$$\begin{array}{l} \log\ 149 = 2.1731863 \\ \log\ 1468 = 3.1667261 \end{array} \qquad \begin{array}{l} \log\ 2134 = 3.3291944 \\ \log\ 815 = 2.9111576 \end{array}$$

$$\left.\begin{array}{r} 5.3399124 \\ 6.2403520 \end{array}\right\} \text{subtract} \qquad 6.2403520$$

$$\begin{array}{r} 2)\bar{1}.0995604 \\ \hline \bar{1}.5497802 \\ 10. \\ \hline \end{array}$$

$$\begin{array}{r} \log \sin \tfrac{1}{2} B = 9.5497802 \\ \log \sin 20^\circ 46' = 9.5496935 \\ \hline 867 = \text{diff.} \end{array}$$

Tab. diff. = 3330

$$\frac{867 \times 60''}{3330} = 15''\tfrac{1}{2} \therefore \tfrac{1}{2} B = 20^\circ\ 46'\ 15''\tfrac{1}{2}$$

and $B = 41^\circ\ 32'\ 31''$. *Ans.*

(4)

$$\begin{array}{r} a = 1500 \\ b = 1342 \\ c = 1110 \\ \hline 2)3952 \\ \hline s = 1976 \\ 1500 \\ \hline s - a = 476 \end{array} \qquad \begin{array}{r} 1976 \\ 1342 \\ \hline s - b = 634 \end{array}$$

$$\log \sin \tfrac{1}{2} C = 10 + \tfrac{1}{2} \{\log (s-a) + \log (s-b) - \log a - \log b\}$$

$$\begin{array}{l} \log 476 = 2.6776070 \\ \log 634 = 2.8020893 \end{array} \qquad \begin{array}{l} \log 1500 = 3.1760913 \\ \log 1342 = 3.1277525 \end{array}$$

$$\begin{array}{r} \left.\begin{array}{r} 5.4796963 \\ 6.3038438 \end{array}\right\} \text{subtract} \\ \hline 2)\bar{1}.1758525 \\ \hline \bar{1}.5879262 \\ 10. \\ \hline \log \sin \tfrac{1}{2} C = 9.5879262 \\ \log \sin 22^\circ\ 46' = 9.5876876 \\ \hline 2386 = \text{diff.} \end{array} \qquad \begin{array}{r} \hline 6.3038438 \end{array}$$

Tab. diff. $= 3009$

$$\frac{2386 \times 60''}{3009} = 47''\tfrac{1}{2} \therefore \tfrac{1}{2} C = 22^\circ\ 46'\ 47''\tfrac{1}{2}$$

and $C = 45^\circ\ 33'\ 35''$. *Ans.*

(5.)

$$a = 1.$$
$$b = 1.32$$
$$c = 0.75$$
$$2)\overline{3.07}$$
$$s = 1.535 \qquad\qquad 1.535$$
$$\underline{1.32} \qquad\qquad \underline{.75}$$
$$s - b = .215 \qquad\qquad s - c = .785$$

$$\log \sin \tfrac{1}{2} A = 10 + \tfrac{1}{2}\{\log(s-b) + \log(s-c) - \log b - \log c\}$$

$$\left.\begin{array}{l}\log .215 = \bar{1}.3324385 \\ \log .785 = \bar{1}.8948697\end{array}\right\}\text{add} \qquad \left.\begin{array}{l}\log 1.32 = 0.1205739 \\ \log .75 = \bar{1}.8750613\end{array}\right\}\text{add}$$

$$\left.\begin{array}{l}\bar{1}.2273082 \\ \bar{1}.9956352\end{array}\right\}\text{subtract} \qquad \bar{1}.9956352$$

$$2)\overline{\bar{1}.2316730}$$
$$\bar{1}.6158365$$
$$\underline{10.}$$
$$\log \sin \tfrac{1}{2} A = 9.6158365$$
$$\log \sin 24^\circ\ 23' = 9.6157812$$
$$553 = \text{diff.}$$

Tab. diff. = 2787

$$\frac{553 \times 60''}{2787} = 12'' \therefore \tfrac{1}{2} A = 24^\circ\ 23'\ 12''$$

and $A = 48^\circ\ 46'\ 24''$. *Ans.*

(6.)

$$\begin{array}{r} a=27 \\ b=32 \\ c=\ \ 9 \\ \hline 2)68 \\ \hline s=34 \\ 27 \\ \hline s-a=\ \ 7 \end{array} \qquad \begin{array}{r} 34 \\ 32 \\ \hline s-b=2 \end{array}$$

$$\log\sin\tfrac{1}{2}C = 10 + \tfrac{1}{2}\{\log(s-a)+\log(s-b)-\log a-\log b\}$$

$$\begin{array}{r} \log 7 = .8450980 \\ \log 2 = .3010300 \\ \hline 1.1461280 \\ 2.9365138 \\ \hline 2)\bar{2}.2096142 \\ \hline \bar{1}.1048071 \\ 10. \\ \hline \end{array} \left.\begin{array}{c} \\ \\ \\ \end{array}\right\} \text{subtract} \qquad \begin{array}{r} \log 27 = 1.4313638 \\ \log 32 = 1.5051500 \\ \hline 2.9365138 \\ \hline \end{array}$$

$$\begin{array}{r} \log\sin\tfrac{1}{2}C = 9.1048071 \\ \log\sin 7^\circ\ 18' = 9.1040246 \\ \hline 7825 = \text{diff.} \end{array}$$

Tab. diff. $= 9850$

$$\frac{7825 \times 60''}{9850} = 47''\tfrac{1}{2} \quad \therefore \tfrac{1}{2}C = 7^\circ\ 18^\circ\ 47''\tfrac{1}{2}$$

and $C = 14^\circ\ 37'\ 35''$. *Ans.*

APPENDIX.

Page 56.

(2.) Mantissa of 85632 = .9326361
(Tab. diff. = 51) × .4 = 20.4   20
Characteristic = 5.

$\therefore$ log 856324 = 5.9326381 *Ans.*

(3.) Mantissa of 17256 = .2369401
(Tab. diff. = 251) × .74 = 185.74   186
Characteristic = 6.

$\therefore$ log 1725674 = 6.2369587 *Ans.*

(4.) Mantissa of 32495 = .5118165
(Tab. diff. = 134) × .678 = 90.852   91
Characteristic = 7.

$\therefore$ log 32495678 = 7.5118256 *Ans.*

(5.) Mantissa of 98765 = .9946031
(Tab. diff. = 44) × .4267 = 18.7748   19
Characteristic = 8.

$\therefore$ log 987654267 = 8.9946050 *Ans.*

(9.) Mantissa of 85673 = .9328440
(Tab. diff. = 51) × .6 = 30.6   31
Characteristic = $\bar{4}$.

$\therefore$ log .000856736 = $\bar{4}$.9328471 *Ans.*

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(2.)

$$\begin{array}{rl} & 1.5673428 \\ \log 36926 = & .5673323 \text{ (omitting the characteristic)} \\ \hline & 105 = \text{Diff.} \\ & 118 = \text{Tab. diff.} \end{array}$$

$\frac{105}{118} = .89$ ∴ Required number $= 36.92689$ *Ans.*

(3.)

$$\begin{array}{rl} & 5.7326421 \\ \log 54030 = & .7326350 \\ \hline & 71 = \text{Diff.} \\ & 80 = \text{Tab. diff.} \end{array}$$

$\frac{71}{80} = .8875$ ∴ Required number $= 540308.875$ *Ans.*

(4.)

$$\begin{array}{rl} & 3.8924652 \\ \log 78066 = & .8924619 \\ \hline & 33 = \text{Diff.} \\ & 56 = \text{Tab. diff.} \end{array}$$

$\frac{33}{56} = .589$ ∴ Required number $= 7806.6589$ *Ans.*

(5.)

$$\begin{array}{rl} & .9276324 \\ \log 84651 = & .9276321 \\ \hline & 3 = \text{Diff.} \\ & 51 = \text{Tab. diff.} \end{array}$$

$\frac{3}{51} = .059$ ∴ Required number $= 8.4651059$ *Ans.*

(6.)

$$\begin{array}{rl} & \bar{2}.5321658 \\ \log 34053 = & .5321554 \\ \hline & 104 = \text{Diff.} \\ & 128 = \text{Tab. diff.} \end{array}$$

$\frac{104}{128} = .8$ ∴ Required number $= .0340538$ *Ans.*

(7.)

$$\begin{array}{rr} & \bar{4}.3215467 \\ \log 20967 = & .3215363 \\ \hline & 104 = \text{Diff.} \\ & 207 = \text{Tab. diff.} \end{array}$$

$\frac{104}{207} = .5$ ∴ Required number $= .000209675$ *Ans.*

(8)

$$\begin{array}{rr} & \bar{1}.1863241 \\ \log 15357 = & .1863064 \\ \hline & 177 = \text{Diff.} \\ & 283 = \text{Tab. diff.} \end{array}$$

$\frac{177}{282} = .62$ ∴ Required number $= .1535762$ *Ans.*

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(4.)

$$\begin{array}{rr} \log 1576.3 = & 3.1976389 \\ \log .85673 = & \bar{1}.9328440 \\ \hline & 3.1304829 \\ \log 13504 = & .1304624 \\ \hline & 205 = \text{Diff.} \\ & 322 = \text{Tab. diff.} \end{array}$$

$\frac{205}{322} = .636$ ∴ Required number $= 1350.4636$ *Ans.*

(5.)

$$\begin{array}{rr} \log 18.54 = & 1.2681097 \\ \log 735.6425 = & 2.8666668 \\ \hline & 4.1347765 \\ \log 13638 = & .1347507 \\ \hline & 258 = \text{Diff.} \\ & 318 = \text{Tab. diff.} \end{array}$$

$\frac{258}{318} = .811$ ∴ Required number $= 13638.811$ *Ans.*

(6.)

$$
\begin{aligned}
\log 21.357 &= 1.3295402\\
\log 6324.567 &= 3.8010308\\
\hline
&\ 5.1305710\\
\log 13507 &= .1305589\\
\hline
121 &= \text{Diff.}\\
322 &= \text{Tab. diff.}
\end{aligned}
$$

$\frac{121}{322} = .376$ ∴ Required number $= 135073.76$ *Ans.*

(7.)

$$
\begin{aligned}
\log 18.21 &= 1.2603099\\
\log 35.672 &= 1.5523275\\
\log 2.847 &= 0.4543875\\
\log 11.256 &= 1.0513841\\
\hline
&\ 4.3184090\\
\log 20816 &= .3183973\\
\hline
117 &= \text{Diff.}\\
209 &= \text{Tab. diff.}
\end{aligned}
$$

$\frac{117}{209} = .56$ ∴ Required number $= 20816.56$ *Ans.*

(8.)

$$
\begin{aligned}
\log .0873 &= \bar{2}.9410142\\
\log 25.773206 &= 1.4111685\\
\log .005693 &= \bar{3}.7553412\\
\hline
&\ \bar{2}.1075239\\
\log 12809 &= \phantom{\bar{2}}.1075152\\
\hline
87 &= \text{Diff.}\\
339 &= \text{Tab. diff.}
\end{aligned}
$$

$\frac{87}{339} = .2$ ∴ Required number $= .0128092$ *Ans.*

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(2.)

$$\log .32567 = \bar{1}.5127778$$
$$\log .0129 = \bar{2}.1105897$$

$$\therefore \log \text{quotient} = 1.4021881$$
$$\log 25245 = .4021754$$

$$127 = \text{Diff.}$$
$$172 = \text{Tab. diff.}$$

$$\frac{127}{172} = .738 \quad \therefore \text{Quotient} = 25.245738 \quad \textit{Ans.}$$

(3.)

$$\log 11067 = .0440299$$
$$(\text{Tab. diff.} = 392) \times .3 = 117.6 \qquad 118$$

$$\therefore \log 110.673 = 2.0440417$$
$$\left.\begin{array}{r} \log 56734 = .7538434 \\ (\text{Tab. diff.} = 77) \times .2 = 15.4 \qquad 15 \end{array}\right\} \text{Retaining only the Mantissa.}$$

$$\therefore \log 567.342 = 2.7538449$$
$$\text{but } \log 110.673 = 2.0440417$$

$$\therefore \log \text{quotient} = .7098032$$
$$\log 51262 = .7097955$$

$$77 = \text{Diff.}$$
$$84 = \text{Tab. diff.}$$

$$\frac{77}{84} = .9 \quad \therefore \text{Quotient} = 5.12629 \quad \textit{Ans.}$$

(4)

$$\log .01237 = \bar{2}.09233697$$
$$\log 108.46 = 2.03532696$$

$$\therefore \log \text{quotient} = \bar{4}.0571001$$
$$\log 11405 = .0570953$$

$$48 = \text{Diff.}$$
$$381 = \text{Tab. diff.}$$

$$\frac{48}{381} = .1 \quad \therefore \text{Quotient} = .000114051 \quad \textit{Ans.}$$

(5) $\log 10056 = .0024253$

(Tab. diff. $= 432) \times .789 = 340.848$ $\quad 341$

$\therefore \log 10056789 = 7.0024594$

$\log 385.27 = 2.5857652$

and $\log 10056789 = 7.0024594$

$\therefore$ log quotient $= \bar{5}.5833058$

$\log 38309 = .5833008$

$50 =$ Diff.

$114 =$ Tab. diff.

$\frac{50}{114} = .44$ $\therefore$ Quotient $= .00003830944$ *Ans.*

(6.) $\log 28764 = .4588493$

(Tab. Diff. $= 151) \times .2 = 30.2$ $\quad 30$

$\therefore \log 287.642 = 2.4588523$

$\log 83456 = 4.9214576$

$\therefore$ log quotient $= \bar{3}.5373947$

$\log 34466 = .5373909$

$38 =$ Diff.

$126 =$ Tab. diff.

$\frac{38}{126} = .3$ $\therefore$ Quotient $= .00344663$ *Ans.*

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(2.) $\log 11 = 1.0413927$

5

5.2069635

$\log 16105 = .2069607$

$28 =$ Diff.

$270 =$ Tab. Diff.

$\frac{28}{270} = .1$ $\therefore$ Required power $= 161051$ *Ans.*

(3.)

$$\begin{array}{rr} \log 82.56 = & 1.9167697 \\ & 2 \\ \hline & 3.8335394 \\ \log 68161 = & .8335360 \\ \hline & 34 = \text{Diff.} \\ & 64 = \text{Tab. diff.} \end{array}$$

$\frac{34}{64} = .53$ ∴ Required power $= 6816.153$ *Ans.*

(4)

$$\begin{array}{rr} \log 196.3 = & 2.2929203 \\ & 3 \\ \hline & 6.8787609 \\ \log 75641 = & .8787573 \\ \hline & 36 = \text{Diff.} \\ & 57 = \text{Tab. diff.} \end{array}$$

$\frac{36}{57} = .63$ ∴ Required power $= 7564163$ *Ans.*

(5.)

$$\begin{array}{rr} \log 1.5 = & \left.\begin{array}{r} .1760913 \\ 21 \end{array}\right\} \text{Multiply} \\ \hline & 3.6979173 \\ \log 49879 = & .6979177 \\ \hline \end{array}$$

∴ Required power $= 4987.9$ *Ans.*

(6.)

$$\begin{array}{rr} \log 1.037 = & \left.\begin{array}{r} .0157788 \\ 297 \end{array}\right\} \text{Multiply} \\ \hline & 4.6863036 \\ \log 48562 = & .6862966 \\ \hline & 70 = \text{Diff.} \\ & 89 = \text{Tab. diff.} \end{array}$$

$\frac{70}{89} = .8$ ∴ Required number $= 48562.8$ *Ans.*

(8.)

$$\begin{array}{rr}
\log .08567 = & \bar{2}.9328288 \\
 & 2 \\
\hline
 & \bar{3}.8656576 \\
\log 73393 = & .8656546 \\
\hline
 & 30 = \text{Diff.} \\
 & 59 = \text{Tab. diff.}
\end{array}$$

$\frac{30}{59} = .5$ ∴ Required power $= .00733935$ *Ans.*

(9.)

$$\begin{array}{rr}
\log .10341 = & \bar{1}.0145625 \\
 & 10 \\
\hline
 & \overline{10}.1456250 \\
\log 13983 = & .1456004 \\
\hline
 & 246 = \text{Diff.} \\
 & 310 = \text{Tab. diff.}
\end{array}$$

$\frac{246}{310} = .79$ ∴ Required number $= .0000000001398379$ *Ans.*

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(2.)

$$\begin{array}{rr}
 & \log 75863 = .8800300 \\
(\text{Tab. diff.} = 57) \times .21 = 11.97 & 12 \\
\hline
\therefore \log 7586.321 = & 3.8800312 \\
 & 2)3.8800312 \\
\hline
 & 1.9400156 \\
\log 87099 = & .9400132 \\
\hline
 & 24 = \text{Diff.} \\
 & 50 = \text{Tab. diff.}
\end{array}$$

$\frac{24}{50} = .48$ ∴ Required root $= 87.09948$ *Ans.*

(3.)

$$\log 1729 = 3.2377950$$

$$3)\underline{3.2377950}$$

$$\begin{aligned} &1.0792650 \\ \log 12002 = &\underline{.0792536} \\ &114 = \text{Diff.} \\ &362 = \text{Tab. diff.} \end{aligned}$$

$\frac{114}{362} = .31$ $\therefore$ Required root $= 12.00231$ *Ans.*

(4.)

$$\log 100000 = 5.0000000$$

$$28)\underline{5.0000000}$$

$$\begin{aligned} &.1787143 \\ \log 15090 = &\underline{.1786892} \\ &251 = \text{Diff.} \\ &288 = \text{Tab. diff.} \end{aligned}$$

$\frac{251}{288} = .87$ $\therefore$ Required root $= 1.509087$ *Ans.*

(5.)

$$\log 1.0653 = .0274719$$

$$35)\underline{.0274719}$$

$$\begin{aligned} &.0007849 \\ \log 10018 = &\underline{.0007810} \\ &39 = \text{Diff.} \\ &434 = \text{Tab. diff.} \end{aligned}$$

$\frac{39}{434} = .09$ $\therefore$ Required root $= 1.001809$ *Ans.*

(8.)

$$\log .1478 = \bar{1}.1696744$$

$$2)\underline{\bar{1}.1696744}$$

$$\begin{aligned} &\bar{1}.5848372 \\ \log 38444 = &\underline{.5848286} \\ &86 = \text{Diff.} \\ &113 = \text{Tab. diff.} \end{aligned}$$

$\frac{86}{113} = .76$ $\therefore$ Required root $= .3844476$ *Ans*

(9.)

$$\begin{array}{rl}
\log .01789 = & \bar{2}.2526103 \\
18) & \bar{2}.2526103 \\
\hline
 & \bar{1}.9029228 \\
\log 79969 = & .9029217 \\
\hline
 & 11 = \text{Diff.} \\
 & 54 = \text{Tab. diff.}
\end{array}$$

$\frac{11}{54} = .2 \therefore$ Required root $= .799692$ *Ans.*

(10.)

$$\begin{array}{rl}
\log 1000000 = & 6.0000000 \\
100) & 6.0000000 \\
\hline
 & .0600000 \\
\log 11481 = & .0599797 \\
\hline
 & 203 = \text{Diff.} \\
 & 378 = \text{Tab. diff.}
\end{array}$$

$\frac{203}{378} = .537 \therefore$ Required root $= 1.1481537$ *Ans.*

Page 64.

(3)

$$\begin{array}{rl}
5 \log 8 = & 4.5154500 \\
3 \log 21 = & 2.6444386 \\
3 \log 56 = & 5.2445640 \\
\hline
\end{array}
\qquad
\begin{array}{rl}
3 \log 48 = & 5.0437236 \\
4 \log 112 = & 8.1968720 \\
\hline
 & 13.2405956
\end{array}$$

$$\begin{array}{rl}
 & \left.\begin{array}{r} 12.4044526 \\ 13.2405956 \end{array}\right\} \text{subtract} \\
\hline
 & \bar{1}.1638570 \\
\log 14583 = & .1638469 \\
\hline
 & 101 = \text{Diff.} \\
 & 298 = \text{Tab. diff.}
\end{array}$$

$\frac{101}{298} = .34 \therefore$ Required value $= .1458334$ *Ans.*

(4.)

$$\begin{array}{rl} \tfrac{1}{5}\log 113 = & 0.4106157 \\ \tfrac{1}{3}\log 8563 = & 1.3108753 \\ \tfrac{1}{2}\log 562 = & 1.3748681 \\ \hline & 3.0963591 \\ & 1.5983358 \\ \hline & 1.4980233 \\ \log 31479 = & .4980209 \\ \hline & 24 = \text{Diff.} \\ & 138 = \text{Tab. diff.} \end{array} \qquad \begin{array}{rl} \tfrac{1}{7}\log 98732 = & 0.7134940 \\ \tfrac{1}{4}\log 3462 = & 0.8848418 \\ \hline & 1.5983358 \end{array}$$

$\left.\begin{array}{r} 3.0963591 \\ 1.5983358 \end{array}\right\}$ subtract

$$\frac{24}{138} = .17 \quad \therefore \text{ Required value} = 31.47917 \quad \textit{Ans.}$$

Page 65.

(3.)

$$\left.\begin{array}{rr} \tan 39^\circ\ 21' = & .8199487 \\ (\text{Tab. diff.} = 4867) \times \dfrac{46''}{60''} = & 3731 \end{array}\right\} \text{add}$$

$$\therefore \tan 39^\circ\ 21'\ 46'' = .8203218 \quad \textit{Ans.}$$

(4.)

$$\left.\begin{array}{rr} \cot 76^\circ\ 53' = & .2330139 \\ (\text{Tab. diff.} = 3066) \times \dfrac{8''}{60''} = & 408 \end{array}\right\} \text{subtract}$$

$$\therefore \cot 76^\circ\ 53'\ 8'' = .2329731 \quad \textit{Ans.}$$

(5.)

$$\left.\begin{array}{rr} \sin 86^\circ\ 3' = & .9976245 \\ (\text{Tab. diff.} = 200) \times \dfrac{17''}{60''} = & 57 \end{array}\right\} \text{add}$$

$$\therefore \sin 86^\circ\ 3'\ 17'' = .9976302 \quad \textit{Ans.}$$

(6.)

$$\left.\begin{array}{rr} \cos 57^\circ\ 32' = & .5368089 \\ (\text{Tab. diff.} = 2454) \times \dfrac{36''}{60''} = & 1472 \end{array}\right\} \text{subtract}$$

$$\therefore \cos 57^\circ\ 32'\ 36'' = .5366617 \quad \textit{Ans.}$$

Page 66.

(3.)

$$\text{Given tan} = 1.5632417$$
$$\tan 57^\circ\ 23' = 1.5626549$$
$$5868 = \text{Diff.}$$
$$10014 = \text{Tab. diff.}$$

$$\frac{5868 \times 60''}{10014} = 35'' \therefore \text{Required angle} = 57^\circ\ 23'\ 35'' \quad \textit{Ans.}$$

(4.)

$$\text{Given cot} = .4763215$$
$$\cot 64^\circ\ 32' = .4762616$$
$$599 = \text{Diff.}$$
$$3569 = \text{Tab. diff.}$$

$$\frac{599 \times 60''}{3569} = 10'' \therefore \text{Required angle} = 64^\circ\ 32' - 10'' = 64^\circ\ 31'\ 50'' \quad \textit{Ans.}$$

(5.)

$$\text{Given sec} = 1.8345672$$
$$\sec 56^\circ\ 58' = 1.8344354$$
$$1318 = \text{Diff.}$$
$$8211 = \text{Tab. diff.}$$

$$\frac{1318 \times 60''}{8211} = 9'' \therefore \text{Required angle} = 56^\circ\ 58'\ 9'' \quad \textit{Ans.}$$

(6.)

$$\text{cosec}\ 27^\circ\ 56' = 2.1347270$$
$$\text{Given cosec} = 2.1346521$$
$$749 = \text{Diff.}$$
$$11701 = \text{Tab. diff.}$$

$$\frac{749 \times 60''}{11701} = 4'' \therefore \text{Required angle} = 27^\circ\ 56' + 4'' = 27^\circ\ 56'\ 4'' \quad \textit{Ans.}$$

Page 68.

(3.)

$$\left.\begin{array}{rr} \log \tan 42^\circ\ 47' = & 9.9663623 \\ (\text{Tab. diff.} = 2534) \times \dfrac{26''}{60} = & 1098 \end{array}\right\} \text{add}$$

$$\therefore \log \tan 42^\circ\ 47'\ 26'' = 9.9664721 \quad \textit{Ans.}$$

(4.)

$$\left.\begin{array}{r} \log\cot 73^\circ\ 21' = 9.4757633 \\ (\text{Tab. diff.} = 4600) \times \dfrac{7''}{60''} = \qquad 537 \end{array}\right\}\text{ subtract}$$

$$\therefore \log\cot 73^\circ\ 21'\ 7'' = 9.4757096 \quad \textit{Ans.}$$

Page 69.

(3.)

$$\begin{array}{r} \text{Given } \log\cos = 9.87324 15 \\ \log\cos 41^\circ\ 4' = 9.8732227 \\ \hline 188 = \text{Diff.} \\ 1125 = \text{Tab. diff.} \end{array}$$

$$\frac{188 \times 60^\circ}{1125} = 10'' \therefore \text{Required angle} = 41^\circ\ 41' - 10'' = 41^\circ\ 40'\ 50'' \quad \textit{Ans.}$$

(4.)

$$\begin{array}{r} \text{Given } \log\tan = 9.7963423 \\ \log\tan 32^\circ\ 1' = 9.7960703 \\ \hline 2720 = \text{Diff.} \\ 2810 = \text{Tab. diff.} \end{array}$$

$$\frac{2720 \times 60''}{2810} = 58'' \therefore \text{Required angle} = 32^\circ\ 1'\ 58'' \quad \textit{Ans.}$$

SOLUTIONS

OF THE

NEW QUESTIONS IN THE FOURTH EDITION OF THE MANUAL OF TRIGONOMETRY.

Pages 5, 6.

(1.) Since $60° = 60 \times 60 \times 60$ seconds, and arc = radius subtends angle at centre = 206265 seconds, we have circular measure of

$$60° = \frac{60 \times 60 \times 60}{206265} = \frac{216000}{206265}$$
$$= 1.04719. \quad \textit{Ans.}$$

(2.) Here circular measure of

$$18° = \frac{18 \times 60 \times 60}{206265} = \frac{64800}{206265}$$
$$= .31416. \quad \textit{Ans.}$$

(3.) Here

$$\angle = \tfrac{3}{4} \text{ of } 206265'' = \frac{618795}{4} = 154698''.75$$
$$= 42° \, 58' \, 18''. \, 75. \quad \textit{Ans.}$$

(4.) Here $\angle = .256 \times 206265'' = 52803''.840$

$$= 14° \, 40' \, 3''.84. \quad \textit{Ans.}$$

(5.) The angular unit has been shown to contain 206265″, but

$$206265'' = 57^\circ\, 17'\, 45''. \quad \textit{Ans.}$$

(6.)
$$\frac{45 \times 60 \times 60}{206265} = \frac{162000}{206265}$$
$$= .78539. \quad \textit{Ans.}$$

Page 16.

(1.)
$$\cos A = \sqrt{(1 - \sin^2 A)} = \sqrt{\{1 - (.42)^2\}}$$
$$= \sqrt{(1 - .1764)} = \sqrt{(.8236)}$$
$$= .907. \quad \textit{Ans.}$$

(2.)
$$\cos A = \frac{1}{\sqrt{(1 + \tan^2 A)}}$$
$$= \frac{1}{\sqrt{(1 + .7^2)}} = \frac{1}{\sqrt{1.49}}$$
$$= \frac{\sqrt{1.49}}{1.49} = \frac{1.220655}{1.49} = .8192. \quad \textit{Ans.}$$

(3.)
$$\sin A = \frac{\tan A}{\sqrt{(1 + \tan^2 A)}} = \frac{\frac{3}{2}}{\sqrt{\left\{1 + \left(\frac{3}{2}\right)^2\right\}}}$$
$$= \frac{3}{\sqrt{(4 + 9)}} = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$$
$$= 3 \times \frac{3.6055}{13} = \frac{10.8165}{13} = .832. \quad \textit{Ans.}$$

(4.) $$\tan A = \frac{\sqrt{\{(1 - \cos^2 A)\}}}{\cos A}$$

$$= \frac{\sqrt{\left\{1 - \left(\frac{5}{9}\right)^2\right\}}}{\frac{5}{9}} = \frac{\sqrt{(81 - 25)}}{5}$$

$$= \frac{\sqrt{56}}{5} = \frac{7.483}{5} = 1.496. \quad \textit{Ans.}$$

(5.) $$\tan A = \frac{\sqrt{(2 \text{ versin } A - \text{versin}^2 A)}}{1 - \text{versin } A}$$

$$= \frac{\sqrt{\left\{2 \times \frac{3}{20} - \left(\frac{3}{20}\right)^2\right\}}}{1 - \frac{3}{20}}$$

$$= \frac{\sqrt{(120 - 9)}}{20 - 3} = \frac{\sqrt{111}}{17} = \frac{10.535}{17}$$

$= .620$ nearly. *Ans.*

(6.) $$\sin A = \frac{\tan A}{\sqrt{(1 + \tan^2 A)}}$$

$$= \frac{.6}{\sqrt{\{1 + (.6)^2\}}} = \frac{.6}{\sqrt{1.36}}$$

$$= \frac{.6 \times \sqrt{1.36}}{1.36} = \frac{.6 \times 1.1662}{1.36}$$

$$= \frac{.69972}{1.36} = .514. \quad \textit{Ans.}$$

(7.)
$$\sin A = \frac{\sqrt{(\sec^2 A - 1)}}{\sec A}$$

$$= \frac{\sqrt{\left\{\left(\frac{23}{20}\right)^2 - 1\right\}}}{\frac{23}{20}} = \frac{\sqrt{(529 - 400)}}{23}$$

$$= \frac{\sqrt{129}}{23} = \frac{11.357}{23} = .494 \text{ nearly.} \quad \textit{Ans.}$$

(8.)
$$\cot A = \frac{\sqrt{(1 - \sin^2 A)}}{\sin A}$$

$$= \frac{\sqrt{\{1 - (.7)^2\}}}{.7} = \frac{\sqrt{(1 - .49)}}{.7}$$

$$= \frac{\sqrt{.51}}{.7} = \frac{.714}{.7} = 1.02. \quad \textit{Ans.}$$

(9.)
$$\sin A = \frac{\tan A}{\sqrt{(1 + \tan^2 A)}}$$

$$= \frac{20}{\sqrt{(1 + 20^2)}} = \frac{20\sqrt{401}}{401}$$

$$= \frac{20 \times 20.02496}{401} = .998. \quad \textit{Ans.}$$

(10.)
$$\sec A = \frac{\sqrt{(1 + \cot^2 A)}}{\cot A}$$

$$= \frac{\sqrt{(1 + 5^2)}}{5} = \frac{\sqrt{26}}{5} = \frac{5.099}{5}$$

$$= 1.019. \quad \textit{Ans.}$$

(11.) $$\tan A = \frac{\sqrt{\{2 \text{ versin } A - \text{versin}^2 A\}}}{1 - \text{versin } A}$$

$$= \frac{\sqrt{\left\{2 \times \frac{1}{9} - \left(\frac{1}{9}\right)^2\right\}}}{1 - \frac{1}{9}} = \frac{\sqrt{(18 - 1)}}{9 - 1}$$

$$= \frac{\sqrt{17}}{8} = \frac{4.123}{8} = .515. \quad \textit{Ans.}$$

(12.) $$\sin A = \sqrt{(1 - \cos^2 A)}$$

$$= \sqrt{\{1 - (1 - \text{versin } A)^2\}}$$

$$= \sqrt{(2 \text{ versin } A - \text{versin}^2 A)}$$

$$= \sqrt{\{2 \times \tfrac{1}{4} - (\tfrac{1}{4})^2\}} = \tfrac{1}{4}\sqrt{(8 - 1)}$$

$$= \tfrac{1}{4}\sqrt{7} = \frac{2.645}{4} = .661. \quad \textit{Ans.}$$

Page 70.

(1.) Here (see Fig., p. 69, Manual),

$$BC = AB \tan CAB = 125 \tan 52^\circ\ 34'$$

$$\therefore \log BC = \log 125 + \log \tan 52^\circ\ 34' - 10$$

$$\log 125 = 2.0969100$$

$$\log \tan 52^\circ\ 34' - 10 = 0.1160662$$

$$\therefore \log BC = 2.2129762$$

$$\therefore BC = 163.296$$

$$\text{and CE} = \text{BC} + \text{BE} = 163.296 + 5.5$$
$$= 168.796 \text{ ft.} \quad \textit{Ans.}$$

(2.) Here (Fig., p. 69, Manual),

$$\text{BC} = \text{AB} \tan \text{CAB} = 42 \tan 49^\circ 28'$$
$$\therefore \log \text{BC} = \log 42 + \log \tan 49^\circ 28' - 10$$
$$\log 42 = 1.6232493$$
$$\log \tan 49^\circ 28' - 10 = \underline{0.0679895}$$
$$\therefore \log \text{BC} = 1.6912388$$
$$\therefore \text{BC} = 49.117$$
$$\text{and CE} = \text{BC} + \text{BE} = 49.117 + 5$$
$$= 54.117 \text{ ft.} \quad \textit{Ans.}$$

Page 71.

(1.) Here (Fig., p. 70, Manual) we have to find FC + BE. Now BE $= 4.5$ ft., B $= 58^\circ 14'$, A $= 36^\circ 42'$, and $\therefore$ B $-$ A $= 21^\circ 32'$, and AB $= 156$ ft.; hence,

$$\text{FC} = \text{AB} \times \frac{\sin \text{A} \sin \text{B}}{\sin (\text{B} - \text{A})}$$
$$= 156 \times \frac{\sin 36^\circ 42' . \sin 58^\circ 14'}{\sin 21^\circ 32'}$$
$$\therefore \log \text{FC} = \log 156 + \log \sin 36^\circ 42'$$
$$+ \log \sin 58^\circ 14' - (\log \sin 21^\circ 32' + 10)$$

$$\left.\begin{aligned}\log 156 &= 2.1931246\\ \log\sin 36^\circ\ 42' &= 9.7764289\\ \log\sin 58^\circ\ 14' &= 9.9295207\end{aligned}\right\}\text{add}$$

$$\left.\begin{aligned}&\ 21.8990742\\ 10+\log\sin 21^\circ\ 32' &= 19.5647163\end{aligned}\right\}\text{subtract}$$

$$\therefore \log FC = 2.3343579$$

$$\therefore FC = 215.952$$

and $FC + BE = 215.952 + 4.5 = 220.452$ ft. *Ans.*

(2.) Here (Fig., p. 70, Manual) we are to find FC.

Now $A = 28^\circ\ 30'$, $B = 52^\circ$, and $\therefore B - A = 23^\circ\ 30'$, and $AB = 900$ ft.

$$FC = AB \times \frac{\sin A \sin B}{\sin(B - A)}$$

$$= 900 \times \frac{\sin 28^\circ\ 30'.\sin 52^\circ}{\sin 23^\circ\ 30'}$$

$$\therefore \log FC = \log 900 + \log\sin 28^\circ\ 30'$$

$$+ \log\sin 52^\circ - (\log\sin 23^\circ\ 30' + 10)$$

$$\left.\begin{aligned}\log 900 &= 2.9542425\\ \log\sin 28^\circ\ 30' &= 9.6786629\\ \log\sin 52^\circ &= 9.8965321\end{aligned}\right\}\text{add}$$

$$\left.\begin{aligned}&\ 22.5294375\\ \log\sin 23^\circ\ 30' + 10 &= 19.6006997\end{aligned}\right\}\text{subtract.}$$

$$\therefore \log FC = 2.9287378$$

$$\therefore FC = 848.668 \text{ ft.} \quad \textit{Ans.}$$

Page 72.

(1.) Here (Fig., p. 71, Manual),

$$AB = \tfrac{1}{4} \text{ of a mile} = 1320 \text{ ft.}$$

$A = 16^\circ\ 28'$, $B = 52^\circ\ 14'$ (and $\therefore B - A = 35^\circ\ 46'$)

$E = 48^\circ\ 38'$, and CD (the height of castle), and DF (its elevation above the sea), are required.

Now (page 70, Manual),

$$FC = AB \times \frac{\sin A \sin B}{\sin (B - A)} \qquad (1)$$

$$FB = AB \times \frac{\sin A \cos B}{\sin (B - A)}$$

Also (p. 71, Manual),

$$DF = FB \,.\, \tan E, \text{ and}$$

$$\therefore DF = AB \times \frac{\sin A \cos B \tan E}{\sin (B - A)} \qquad (2)$$

From (1) and (2) we have—

$$\log FC = \log AB + \log \sin A + \log \sin B - \{\log \sin (B - A) + 10\} \qquad (3)$$

$$\log DF = \log AB + \log \sin A + \log \cos B + \log \tan E - \{\log \sin (B - A) + 20\} \qquad (4)$$

$\therefore$ by substitution (3) and (4) become

$$\log FC = \log 1320 + \log \sin 16^\circ\ 28' + \log \sin 52^\circ\ 14' - (\log \sin 35^\circ 46' + 10),$$

and $$\log DF = \log 1320 + \log \sin 16^\circ\ 28' + \log \cos 52^\circ\ 14' + \log \tan 48^\circ\ 38'$$

$$-(\log \sin 35^\circ 46' + 20)$$

$$\left.\begin{aligned}\log 1320 &= 3.1205739\\ \log \sin 16^\circ 28' &= 9.4524879\\ \log \sin 52^\circ 14' &= 9.8979082\end{aligned}\right\}\text{ add}$$

$$\left.\begin{aligned}&22.4709700\\ \log \sin 35^\circ 46' + 10 = {}&19.7667739\end{aligned}\right\}\text{ subtract}$$

$$\therefore \log FC = 2.7041961$$

$$\therefore FC = 506.053 \text{ ft.}$$

$$\left.\begin{aligned}\log 1320 &= 3.1205739\\ \log \sin 16^\circ 28' &= 9.4524879\\ \log \cos 52^\circ 14' &= 9.7870687\\ \log \tan 48^\circ 38' &= 10.0552286\end{aligned}\right\}\text{ add}$$

$$\left.\begin{aligned}&32.4153591\\ \log \sin 35^\circ 46' + 20 = {}&29.7667739\end{aligned}\right\}\text{ subtract}$$

$$\therefore \log DF = 2.6485852$$

$\therefore$ DF = 445.23 ft., the required height above the sea.

Hence,

$$\text{height of castle} = FC - DF = 506.05 - 445.23 = 60.82 \text{ ft.} \quad \textit{Ans.}$$

(2.) By (3) and (4) of last question, we have—

$$\log FC = \log AB + \log \sin A + \log \sin B - \{\log \sin (B - A) + 10\} \text{ and}$$

$$\log DF = \log AB + \log \sin A + \log \cos B - \{\log \sin (B - A) + 20\}$$

or, by substitution,

$$\log FC = \log 54 + \log \sin 31^\circ 30' + \log \sin 48^\circ - (\log \sin 16^\circ 30' + 10)$$

and $\log DF = \log 54 + \log \sin 31^\circ 30' + \log \cos 48^\circ + \log \tan 36^\circ 30' - \{\log \sin 16^\circ 30' + 20\}.$

$$\left.\begin{aligned} \log 54 &= 1.7323938 \\ \log \sin 31^\circ 30' &= 9.7180851 \\ \log \sin 48^\circ\ 0' &= 9.8710735 \end{aligned}\right\} \text{add}$$

$$\left.\begin{aligned} &21.3215524 \\ \log \sin 16^\circ 30' + 10 = &19.4533418 \end{aligned}\right\} \text{subtract}$$

$$\therefore \log FC = 1.8682106$$

$$\therefore FC = 73.826 \text{ ft.}$$

$$\left.\begin{aligned} \log 54 &= 1.7323938 \\ \log \sin 31^\circ 30' &= 9.7180851 \\ \log \cos 48^\circ\ 0' &= 9.8255109 \\ \log \tan 36^\circ 30' &= 9.8692089 \end{aligned}\right\} \text{add}$$

$$\left.\begin{aligned} &31.1451987 \\ \log \sin 16^\circ 30' + 20 = &29.4533418 \end{aligned}\right\} \text{subtract}$$

$$\therefore \log DF = 1.6918569$$

and $DF = 49.187$ ft. = height above the ground;

and $FC - DF = 73.826 - 49.187$
$= 24.639$ ft. = height of window.

Page 73.

(1.) Here $h = 30$ feet, $d = 15^\circ 40'$, $d' = 10^\circ$; and it is required to find FC (the distance of the object), and $AF + h$ (the height of the house).

$$\text{Now } FC = h \times \frac{\cos d \cos d'}{\sin (d - d')}$$

$$= 30 \times \frac{\cos 15^\circ 40' . \cos 10^\circ}{\sin 5^\circ 40'}$$

$$\text{and AF} = 30 \times \frac{\cos 15^\circ 40' \sin 10^\circ}{\sin 5^\circ 40'}$$

$$\therefore \log \text{FC} = \log 30 + \log \cos 15^\circ 40' + \log \cos 10^\circ$$
$$- (\log \sin 5^\circ 40' + 10);$$

$$\text{and} \quad \log \text{AF} = \log 30 + \log \cos 15^\circ 40' + \log \sin 10^\circ$$
$$- (\log \sin 5^\circ 40' + 10)$$

$$\left.\begin{aligned} \log 30 &= 1.4771213 \\ \log \cos 15^\circ 40' &= 9.9835582 \\ \log \cos 10^\circ &= 9.9933515 \end{aligned}\right\} \text{add}$$

$$\left.\begin{aligned} & 21.4540310 \\ \log \sin 5^\circ 40' + 10 &= 18.9944968 \end{aligned}\right\} \text{subtract}$$

$$\therefore \log \text{FC} = 2.4595342$$

$$\therefore \text{FC} = 288.094 \text{ feet} = \text{distance of object.}$$

$$\left.\begin{aligned} \log 30 &= 1.4771213 \\ \log \cos 15^\circ 40' &= 9.9835582 \\ \log \sin 10^\circ &= 9.2396702 \end{aligned}\right\} \text{add}$$

$$\left.\begin{aligned} & 20.7003497 \\ \log \sin 5^\circ 40' + 10 &= 18.9944968 \end{aligned}\right\} \text{subtract}$$

$$\therefore \log \text{AF} = 1.7058529$$

$$\therefore \text{AF} = 50.7987$$

$$\therefore \text{AF} + h = 50.7987 + 30 = 80.7987 \text{ feet}$$
$$= \text{height of house.}$$

(2.) Here $h = 68$ feet,

$$d = 16^\circ 28', d' = 14^\circ 21'$$

and $\therefore d - d' = 2^\circ\ 7'$.

Now FC (distance required)

$$= h \times \frac{\cos d . \cos d'}{\sin (d - d')} = 68 \times \frac{\cos 16^\circ\ 28' . \cos 14^\circ\ 21'}{\sin 2^\circ\ 7'}$$

$$\therefore \log FC = \log 68 + \log \cos 16^\circ\ 28'$$

$$+ \log \cos 14^\circ\ 21' - (\log \sin 2^\circ\ 7' + 10)$$

$$\left.\begin{aligned} \log 68 &= 1.8325089 \\ \log \cos 16^\circ\ 28' &= 9.9818117 \\ \log \cos 14^\circ\ 21' &= 9.9862340 \end{aligned}\right\} \text{add}$$

$$\left.\begin{aligned} &21.8005546 \\ 10 + \log \sin 2^\circ\ 7' = {} &18.5674310 \end{aligned}\right\} \text{subtract}$$

$$\therefore \log FC = 3.2331236$$

$\therefore$ FC = 1710.4 feet = 570.1 yards. *Ans.*

Page 74.

(1.) Let (Fig., p. 73, Manual) C be the windmill, A the station whose distance is required, and B the flagstaff; then AB = 356 yards, $\angle CAB = 53^\circ\ 4'$, $\angle CBA = 49^\circ\ 10'$ (and $\therefore \angle ACB = 180^\circ - 53^\circ\ 4' - 49^\circ\ 10' = 77^\circ\ 46'$), and it is required to find AC.

$$\text{Now } \frac{AC}{AB} = \frac{\sin CBA}{\sin ACB} = \frac{\sin 49^\circ\ 10'}{\sin 77^\circ\ 46'}$$

$$\therefore AC = 356 \times \frac{\sin 49^\circ\ 10'}{\sin 77^\circ\ 46'}$$

$$\therefore \log AC = \log 356 + \log \sin 49^\circ\ 10' - \log \sin 77^\circ\ 46'$$

$$\left.\begin{array}{r} \log 356 = 2.5514500 \\ \log \sin 49^\circ\ 10' = 9.8788748 \end{array}\right\} \text{add}$$

$$\left.\begin{array}{r} 12.4303248 \\ \log \sin 77^\circ\ 46' = 9.9900247 \end{array}\right\} \text{subtract}$$

$$\therefore \log AC = 2.4403001$$

$$\therefore AC = 275.613 \text{ yards. } \textit{Ans.}$$

(2.) Let (Fig., p. 73, Manual) A be the picket, C the Redan, and B the second station; then AB = 200 paces, $\angle CAB = 90^\circ$, and $\angle ABC = 67^\circ\ 23'$. Now from Δ ACB right-angled at A we have AC = AB tan ABC $= 200 \tan 67^\circ\ 23'$.

$$\therefore \log AC = \log 200 + \log \tan 67^\circ\ 23' - 10$$

$$= 2.3010300 + 0.3802795 = 2.6813095.$$

$$\therefore AC = 480.075 \text{ paces. } \textit{Ans.}$$

(1 *bis.*) Here (Fig., p. 74, Manual) b (= AC) = 300 yards, a (= BC) = 450 yards, and $\angle C = 58^\circ\ 20'\ 30''$ $\therefore \frac{1}{2}(A + B)$

$$= 90^\circ - \frac{58^\circ\ 20'\ 30''}{2} = 90^\circ - 29^\circ\ 10'\ 15''$$

$$= 60^\circ\ 49'\ 45''.$$

$$\text{Now, } \frac{\tan \frac{1}{2}(A - B)}{\tan \frac{1}{2}(A + B)} = \frac{a - b}{a + b} = \frac{450 - 300}{450 + 300}$$

$$= \frac{150}{750} = \frac{1}{5} = .2$$

$$\therefore \tan \tfrac{1}{2}(A - B) = .2 \times \tan 60^\circ\ 49'\ 45''$$

$$\therefore \log \tan \tfrac{1}{2}(A - B) = \log .2 + \log \tan 60^\circ\ 49'\ 45''$$

$$\left.\begin{aligned} \log .2 &= \bar{1}.3010300 \\ \log \tan 60^\circ\ 49'\ 45'' &= 10.2531998 \end{aligned}\right\} \text{add}$$

$$\therefore \log \tan \tfrac{1}{2}(A - B) = 9.5542298$$

$$\begin{aligned} \therefore \tfrac{1}{2}(A - B) &= 19^\circ\ 42'\ 43'' \\ \text{but } \tfrac{1}{2}(A + B) &= 60^\circ\ 49'\ 45'' \\ \therefore A &= 80^\circ\ 32'\ 28'' \\ \text{and } B &= 41^\circ\ 7'\ 2''. \end{aligned}$$

Again, $$\frac{c}{b} = \frac{\sin C}{\sin B}$$

$$\therefore c = b\,\frac{\sin C}{\sin B} = \frac{300 \times \sin 58^\circ\ 20'\ 30''}{\sin 41^\circ\ 7'\ 2''}$$

$$\therefore \log c \text{ (or AB)} = \log 300$$

$$+ \log \sin 58^\circ\ 20'\ 30'' - \log \sin 41^\circ\ 7'\ 2''$$

$$\left.\begin{aligned} \log 300 &= 2.4771213 \\ \log \sin 58^\circ\ 20'\ 30'' &= 9.9300280 \end{aligned}\right\} \text{add}$$

$$\left.\begin{aligned} & 12.4071493 \\ \log \sin 41^\circ\ 7'\ 2'' &= 9.8179629 \end{aligned}\right\} \text{subtract}$$

$$\therefore \log AB = 2.5891864$$

$$\therefore AB = 388.317 \text{ yards.} \quad \textit{Ans.}$$

Page 75.

(1.) Here $a = 3$ miles 88 yards $= 5368$ yards,

$b = 2$ miles 560 yards $= 4080$ yards,

and $C = 54^\circ\ 32'\ 40''$.

$$\text{Now } \cos\phi = \frac{2\cos\frac{1}{2}C\sqrt{(ab)}}{a+b} \qquad (\alpha)$$

$$\text{and } c = (a+b)\sin\phi. \qquad (\beta)$$

From (α) and (β), by taking logarithms, we have,

$$\log\cos\phi = \log 2 + \log\cos\tfrac{1}{2}C + \tfrac{1}{2}(\log a + \log b)$$
$$- \log(a+b), \qquad (\gamma)$$

$$\text{and } \log c = \log(a+b) + \log\sin\phi - 10. \qquad (\delta)$$

To compute (γ), we proceed thus—

$$\left.\begin{aligned}\log a &= \log 5368 = 3.7298125\\ \log b &= \log 4080 = 3.6106602\end{aligned}\right\}\text{ add}$$

$$2)7.3404727$$

$$\left.\begin{aligned}\therefore \tfrac{1}{2}(\log a + \log b) &= 3.6702363\\ \log 2 &= 0.3010300\\ \log\cos\tfrac{1}{2}C = \log\cos 27^\circ 16' 20'' &= 9.9488233\end{aligned}\right\}\text{ add}$$

$$\left.\begin{aligned}&13.9200896\\ \log(a+b) = \log 9448 = {}&3.9753399\end{aligned}\right\}\text{ subtract}$$

$$\therefore \log\cos\phi = 9.9447497$$

$$\therefore \phi = 28^\circ\ 17'\ 32''.$$

Again, to compute c, the required distance,

$$\left.\begin{aligned}\log(a+b) &= \log 9448 = 3.9753399\\ \log\sin\phi &= \log\sin 28^\circ\ 17'\ 32'' = 9.6757496\end{aligned}\right\}\text{ add}$$

$$13.6510895$$
$$10.$$

$$\therefore \log c = 3.6510895$$

$\therefore c = 4478.05$ yards $= 2$ miles 958.05 yards. *Ans.*

Page 76.

(1.) In the $\triangle ACB$ (Fig., p. 76, Manual) we have given $AB = 1000$ yards, $\angle BAC = 76^\circ\ 30'$, $\angle ABC = 46^\circ\ 5'$, and $\therefore \angle ACB = 180^\circ - 76^\circ\ 30' - 46^\circ\ 5' = 180^\circ - 122^\circ\ 35' = 57^\circ\ 25'$; to find AC.

$$\text{Now } \frac{AC}{AB} = \frac{\sin ABC}{\sin ACB} \therefore AC = AB \cdot \frac{\sin ABC}{\sin ACB}$$

$$= 1000 \times \frac{\sin 46^\circ\ 5'}{\sin 57^\circ\ 25'} \therefore \log AC = \log 1000$$

$$+ \log \sin 46^\circ\ 5' - \log \sin 57^\circ\ 25'.$$

$$\left.\begin{aligned} \log 1000 &= 3.0000000 \\ \log \sin 46^\circ\ 5' &= 9.8575432 \end{aligned}\right\} \text{add}$$

$$\left.\begin{aligned} &\ 12.8575432 \\ \log \sin 57^\circ\ 25' &= 9.9256261 \end{aligned}\right\} \text{subtract}$$

$$\therefore \log AC = 2.9319171$$

$$\therefore AC = 854.903.$$

Again, in the $\triangle ABD$ we have $AB = 1000$ yards, $\angle BAD = 44^\circ\ 10'$, $ABD = 81^\circ\ 12'$, and $\therefore \angle ADB = 180^\circ - 44^\circ\ 10' - 81^\circ\ 12' = 180^\circ - 125^\circ\ 22' = 54^\circ\ 38'$; to find AD.

$$\frac{AD}{AB} = \frac{\sin ABD}{\sin ADB} \therefore AD = AB \cdot \frac{\sin ABD}{\sin ADB}$$

$$= 1000 \times \frac{\sin 81^\circ\ 12'}{\sin 54^\circ\ 38'}$$

$$\therefore \log AD = \log 1000 + \log \sin 81^\circ\ 12' - \log \sin 54^\circ\ 38'$$

$$\left.\begin{array}{r} \log 1000 = 3.0000000 \\ \log \sin 81^\circ\ 12' = 9.9948573 \end{array}\right\} \text{add}$$

$$\left.\begin{array}{r} 12.9948573 \\ \log \sin 54^\circ\ 38' = \ \ 9.9114051 \end{array}\right\} \text{subtract}$$

$$\therefore \log AD = \ \ 3.0834522$$

$$\therefore AD = 1211.86$$

Lastly, in the Δ CAD we have the two sides AC $= 854.9$, and AD $= 1211.86$, and the contained $\angle$ CAD $=$ CAB $-$ BAD $= 76^\circ\ 30' - 44^\circ\ 10' = 32^\circ\ 20'$; to find CD.

By the formulæ, page 75, Manual, we have—

$$\cos \phi = \frac{2 \cos \frac{1}{2} \text{CAD} \sqrt{(AC \times AD)}}{AC + AD}$$

$$= \frac{2 \cos 16^\circ\ 10' \sqrt{(AC \times AD)}}{854.9 + 1211.86},$$

$$= \frac{2 \cos 16^\circ\ 10' \sqrt{(AC \times AD)}}{2066.76}$$

and $\quad CD = (AC + AD) \sin \phi = 2066.76 \sin \phi;$

hence $\quad \log \cos \phi = \log 2 + \log \cos 16^\circ\ 10' + \frac{1}{2}$
$(\log AC + \log AD) - \log 2066.76$

$$\left.\begin{array}{r} \log AC = 2.9319171 \\ \log AD = 3.0834522 \end{array}\right\} \text{add}$$

$$2)6.0153693$$

$$\left.\begin{array}{r} 3.0076846 \\ \log 2 = 0.3010300 \\ \log \cos 16^\circ\ 10' = 9.9824774 \end{array}\right\} \text{add}$$

$$\left.\begin{array}{r} 13.2911920 \\ \log 2066.76 = 3.3152900 \end{array}\right\} \text{subtract}$$

$$\therefore \log \cos \phi = 9.9759020$$

$$\therefore \phi = 18^\circ\ 54'\ 39''.$$

Also, $\log CD = \log 2066.76 + \log \sin \phi - 10$

$= \log 2066.76 + \log \sin 18^\circ\ 54'\ 39'' - 10$

$= 3.3152900 + 9.5106740 - 10 = 2.8259640$

$\therefore CD = 669.83$ yards. *Ans.*

(2.) Let C and D be the two Redans (Fig., p. 76, Manual), and AB the base line, then $\angle BAC = 118^\circ\ 20'$, $\angle BAD = 46^\circ\ 14'$, and $\therefore \angle CAD = 118^\circ\ 20' - 46^\circ\ 14' = 72^\circ\ 6'$, $\angle ABD = 88^\circ\ 48'$, $\angle ABC = 33^\circ\ 12'$, $\therefore \angle ACB = 180^\circ - 118^\circ\ 20' - 33^\circ\ 12' = 180^\circ - 151^\circ\ 32' = 28^\circ\ 28'$ and $\angle ADB = 180^\circ - 88^\circ\ 48' - 46^\circ\ 14' = 180^\circ - 135^\circ\ 2' = 44^\circ\ 58'$.

$$\text{Now } \frac{AC}{AB} = \frac{\sin ABC}{\sin ACB} \therefore AC = AB \cdot \frac{\sin ABC}{\sin ACB}$$

$$= 500 \times \frac{\sin 33^\circ\ 12'}{\sin 28^\circ\ 28'}$$

$$\therefore \log AC = \log 500 + \log \sin 33^\circ\ 12' - \log \sin 28^\circ\ 28'$$

$$\left.\begin{array}{r} \log 500 = 2.6989700 \\ \log \sin 33^\circ\ 12' = 9.7384343 \end{array}\right\} \text{add}$$

$$\left.\begin{array}{r} 12.4374043 \\ \log \sin 28^\circ\ 28' = 9.6781972 \end{array}\right\} \text{subtract}$$

$$\therefore \log AC = 2.7592071$$

$$\therefore AC = 574.39.$$

$$\text{Again, } AD = AB \cdot \frac{\sin ABD}{\sin ADB} = 500 \times \frac{\sin 88^\circ\ 48'}{\sin 44^\circ\ 58'}$$

$$\therefore \log AD = \log 500 + \log \sin 88^\circ\ 48' - \log \sin 44^\circ\ 58'$$

$$\left.\begin{aligned}\log 500 &= 2.6989700\\ \log\sin 88^\circ 48' &= 9.9999047\end{aligned}\right\}\text{ add}$$

$$\left.\begin{aligned}&12.6988747\\ \log\sin 44^\circ 58' = {}&\ 9.8492322\end{aligned}\right\}\text{ subtract}$$

$$\therefore \log AD = 2.8496425$$

$$\therefore AD = 707.36.$$

Lastly, from Δ CAD, by the formulæ of page 75, Manual, we have—

$$\cos\phi = \frac{2\cos\frac{1}{2}CAD\ \sqrt{(AC \times AD)}}{AC + AD}$$

$$= \frac{2\cos 36^\circ 3'\ \sqrt{(AC \times AD)}}{574.39 + 707.36}$$

$$= \frac{2\cos 36^\circ 3'\ \sqrt{(AC \times AD)}}{1281.75}$$

$$\text{and } CD = (AB + AD)\sin\phi;$$

$$\therefore \log\cos\phi = \log 2 + \log\cos 36^\circ 3' + \tfrac{1}{2}(\log AC + \log AD)$$
$$- \log 1281.75$$

$$\left.\begin{aligned}\log AC &= 2.7592071\\ \log AD &= 2.8496425\end{aligned}\right\}\text{ add}$$

$$2)5.6088496$$

$$\left.\begin{aligned}&2.8044248\\ \log 2 &= 0.3010300\\ \log\cos 36^\circ 3' &= 9.9076820\end{aligned}\right\}\text{ add}$$

$$\left.\begin{aligned}&13.0131368\\ \log 1281.75 &= 3.1078034\end{aligned}\right\}\text{ subtract}$$

$$\therefore \log\cos\phi = 9.9053334$$

$$\therefore \phi = 36^\circ\ 28'\ 20''$$

hence $\log CD = \log(AC + AD) + \log\sin\phi - 10$

$$= \log 1281.75 + \log\sin 36^\circ\ 28'\ 20'' - 10$$

$$= 3.1078034 + 9.7741029 - 10 = 2.8819063$$

$$\therefore CD \doteq 761.91 \text{ yards.} \quad \textit{Ans.}$$

Page 78.

(1.) Here $\angle BAE = BDC = 22^\circ\ 10'$, $\angle ABE = ADC = 21^\circ$, and $AB = 4$ miles; hence,

$$AE = AB \cdot \frac{\sin ABE}{\sin AEB} = AB \cdot \frac{\sin ABE}{\sin(ABE + BAE)}$$

$$= 4 \times \frac{\sin 21^\circ}{\sin 43^\circ\ 10'} \text{ and } BE = AB \cdot \frac{\sin BAE}{\sin(ABE + BAE)}$$

$$= 4 \times \frac{\sin 22^\circ\ 10'}{\sin 43^\circ\ 10'}.$$

$$\therefore \log AE = \log 4 + \log\sin 21^\circ - \log\sin 43^\circ\ 10'$$

and $\log BE = \log 4 + \log\sin 22^\circ\ 10' - \log\sin 43^\circ\ 10'$

$$\left.\begin{array}{r} \log 4 = 0.6020600 \\ \log\sin 21^\circ = 9.5543292 \end{array}\right\} \text{add}$$

$$\left.\begin{array}{r} 10.1563892 \\ \log\sin 43^\circ\ 10' = 9.8351341 \end{array}\right\} \text{subtract}$$

$$\therefore \log AE = 0.3212551$$

$$\therefore AE = 2.0953 \text{ miles.}$$

$$\left.\begin{aligned} \log 4 &= 0.6020600 \\ \log \sin 22^\circ\, 10' &= 9.5766892 \end{aligned}\right\} \text{add}$$

$$\left.\begin{aligned} &\underline{10.1787492} \\ \log \sin 43^\circ\, 10' &= \underline{9.8351341} \end{aligned}\right\} \text{subtract}$$

$$\therefore \log BE = 0.3436151$$

$$\therefore BE = 2.206 \text{ miles.}$$

Again, in Δ ABC we have

$$AB = 4 \text{ miles, } BC = 3, \text{ and } CA = 5.$$

Now $CA^2 = 5^2 = 25 = 4^2 + 3^2 = AB^2 + BC^2$, $\therefore$ the $\angle CBA = 90^\circ$; hence $CB = CA \sin CAB$,

$$\therefore \log \sin CAB = \log CB - \log CA + 10$$

$$= \log 3 - \log 5 + 10 = .4771213 - .6989700 + 10$$

$$= 9.7781513 \therefore \angle CAB = 36^\circ\, 52'\, 11'',$$

$$\therefore \angle ACB = 90^\circ - 36^\circ\, 52'\, 11'' = 53^\circ\, 7'\, 49'';$$

$$\text{hence the } \angle EAC = CAB - BAE = 36^\circ\, 52'\, 11''$$

$$- 22^\circ\, 10' = 14^\circ\, 42'\, 11''.$$

Hence in Δ ACE we have $\angle EAC = 14^\circ\, 42'\, 11''$ (and $\therefore AEC + ACE = 180^\circ - 14^\circ\, 42'\, 11'' = 165^\circ\, 17'\, 49''$).

$$AC = 5 \text{ miles and } AE = 2.0953 \text{ miles.}$$

$$\text{Now } \frac{AC - AE}{AC + AE} = \frac{\tan \frac{1}{2}(AEC - ACE)}{\tan \frac{1}{2}(AEC + ACE)}$$

that is,
$$\frac{2.9047}{7.0953} = \frac{\tan \frac{1}{2}(AEC - ACE)}{\tan 82^\circ\, 38'\, 54''}$$

$$\therefore \log \tan \tfrac{1}{2}(AEC - ACE) = \log \tan 82^\circ\, 38'\, 54''$$

$$+ \log 2.9047 - \log 7.0953.$$

$$\left.\begin{aligned} \log \tan 82^\circ\ 38'\ 54'' &= 10.8893444 \\ \log 2.9047 &= 0.4631013 \end{aligned}\right\} \text{add}$$

$$\left.\begin{aligned} &\ 11.3524457 \\ \log 7.0953 &= 0.8509708 \end{aligned}\right\} \text{subtract}$$

$$\therefore \log \tan \tfrac{1}{2}(\text{AEC} - \text{ACE}) = 10.5014749$$

$$\therefore \tfrac{1}{2}(\text{AEC} - \text{ACE}) = 72^\circ\ 30'\ 27'',$$

$$\text{but } \tfrac{1}{2}(\text{AEC} + \text{ACE}) = 82^\circ\ 38'\ 54'',$$

$$\therefore \angle \text{AEC} = 155^\circ\ 9'\ 21''$$

$$\text{and } \angle \text{ACE} = 10^\circ\ 8'\ 27''.$$

Hence,

$$\angle \text{BCE} = \text{ACB} - \text{ACE} = 53^\circ\ 7'\ 49'' - 10^\circ\ 8'\ 27''$$

$$= 42^\circ\ 59'\ 22''.$$

In the Δ^s ADC and BCD we now have

$$\text{AC} = 5 \text{ miles}, \angle \text{ACD} = 10^\circ\ 8'\ 27'', \angle \text{ADC} = 21^\circ,$$

$$\angle \text{BDC} = 22^\circ\ 10', \angle \text{BCD} = 42^\circ\ 59'\ 22'',$$

$$\text{and BC} = 3 \text{ miles}.$$

Hence,

$$\text{AD} = \text{AC}\,.\frac{\sin \text{ACD}}{\sin \text{ADC}} = 5 \times \frac{\sin 10^\circ\ 8'\ 27''}{\sin 21^\circ}$$

$$\text{CD} = \text{AC}\,.\frac{\sin(\text{ACD} + \text{ADC})}{\sin \text{ADC}} = 5 \times \frac{\sin 31^\circ\ 8'\ 27''}{\sin 21^\circ}$$

$$\text{BD} = \text{BC}\,.\frac{\sin \text{BCD}}{\sin \text{BDC}} = 3 \times \frac{\sin 42^\circ\ 59'\ 22''}{\sin 22^\circ\ 10'}$$

$$\left.\begin{array}{r} \log 5 = 0.6989700 \\ \log \sin 10^\circ\ 8'\ 27'' = 9.2456810 \end{array}\right\} \text{add}$$

$$\left.\begin{array}{r} 9.9446510 \\ \log \sin 21^\circ = 9.5543292 \end{array}\right\} \text{subtract}$$

$$\therefore \log \text{AD} = 0.3903218$$

$$\therefore \text{AD} = 2.4565 \text{ miles.}$$

$$\left.\begin{array}{r} \log 5 = 0.6989700 \\ \log \sin 31^\circ\ 8'\ 27'' = 9.7136110 \end{array}\right\} \text{add}$$

$$\left.\begin{array}{r} 10.4125810 \\ \log \sin 21^\circ = 9.5543292 \end{array}\right\} \text{subtract}$$

$$\therefore \log \text{CD} = 0.8582518$$

$$\therefore \text{CD} = 7.2153 \text{ miles.}$$

$$\left.\begin{array}{r} \log 3 = 0.4771213 \\ \log \sin 42^\circ\ 59'\ 22'' = 9.8336975 \end{array}\right\} \text{add}$$

$$\left.\begin{array}{r} 10.3108188 \\ \log \sin 22^\circ\ 10' = 9.5766892 \end{array}\right\} \text{subtract}$$

$$\therefore \log \text{BD} = 0.7341296$$

$$\therefore \text{BD} = 5.4216 \text{ miles.}$$

Page 79.

(1.) Here $a = \text{BC} = 1300$ feet, $b = \text{AC} = 750$ feet, and $c = \text{AB} = \text{AC} + \text{BC} = 750 + 1300 = 2050$ feet.

$$\text{Now } h^2 = \frac{abc}{a \cot^2 \text{A} - c \cot^2 \text{C} + b \cot^2 \text{B}}.$$

and we may compute the denominator of this fraction as follows:—

$\log a \cot^2 A = \log a + 2 \log \cot A - 20$ in common (*not tabular*) logarithms.

$$
\begin{array}{rr}
\log \cot A = \log \cot 50^\circ\ 20' = & 9.9186769 \\
 & 2 \\
\hline
 & 19.8373538 \\
\log a = \log 1300 = & 3.1139434 \\
\hline
 & 22.9512972 \\
 & 20 \\
\hline
\therefore \log a \cot^2 A = & 2.9512972
\end{array}
$$

$$\therefore a \cot^2 A = 893.927$$

$$
\begin{array}{rr}
\log \cot C = \log \cot 58^\circ\ 14' = & 9.7918458 \\
 & 2 \\
\hline
 & 19.5836916 \\
\log c = \log 2050 = & 3.3117539 \\
\hline
 & 22.8954455 \\
 & 20 \\
\hline
\therefore \log c \cot^2 C = & 2.8954455
\end{array}
$$

$$\therefore c \cot^2 C = 786.041$$

$$
\begin{array}{rr}
\log \cot B = \log \cot 49^\circ\ 25' = & 9.9327777 \\
 & 2 \\
\hline
 & 19.8655554 \\
\log b = \log 750 = & 2.8750613 \\
\hline
 & 22.7406167 \\
 & 20 \\
\hline
\therefore \log b \cot^2 B = & 2.7406167
\end{array}
$$

$$\therefore b \cot^2 B = 550.321$$

Hence,

$$\left.\begin{aligned} a \cot^2 A &= 893.927 \\ b \cot^2 B &= 550.321 \end{aligned}\right\} \text{add}$$

$$\left.\begin{aligned} &1444.248 \\ c \cot^2 C = {} &786.041 \end{aligned}\right\} \text{subtract}$$

$$658.207$$

$$\therefore h^2 = \frac{abc}{658.207} = \frac{1300 \times 750 \times 2050}{658.207}$$

$$\therefore 2 \log h = \log 1300 + \log 750 + \log 2050 - \log 658.207$$

$$\left.\begin{aligned} \log 1300 &= 3.1139434 \\ \log 750 &= 2.8750613 \\ \log 2050 &= 3.3117539 \end{aligned}\right\} \text{add}$$

$$\left.\begin{aligned} &9.3007586 \\ \log 658.207 = {} &2.8183625 \end{aligned}\right\} \text{subtract}$$

$$2)6.4823961$$

$$\therefore \log h = 3.2411980$$

$$\therefore h = 1742.6 \text{ feet.} \quad \textit{Ans.}$$

Pages 82 to 88 (both inclusive).

(1.) Here, since $10'$ is very small, the height of the man (6 feet) may be considered as the arc of a circle with radius = distance of man; hence, since arc = radius subtends at centre an $\angle = 206265''$, we have the following proportion:—

$(10' =)\ 600'' : 206265'' :: 6$ feet : d (distance of man),

$$\therefore d = \frac{206265 \times 6}{600} = \frac{206265}{100} = 2062.65 \text{ feet.} \quad \textit{Ans.}$$

(2.) Here, as in the last question, we find the distance d by the following proportion:—

$$(12' =)\ 720'' : 206265'' :: 5 \text{ feet} : d,$$

$$\therefore d = \frac{206265 \times 5}{720} = \frac{206265}{144} = 1432.39 \text{ feet.} \quad \textit{Ans.}$$

(3.) $(4' =)\ 240'' : 206265'' :: 8$ feet : d,

$$\therefore d = \frac{206265 \times 8}{240} = \frac{206265}{30} = 6875.5 \text{ feet}$$

$$= 1.3 \text{ mile.} \quad \textit{Ans.}$$

(4.) (Fig., p. 72, Manual.)

Let C be the ship's place, A the summit of the cliff AF, and AB = 24 feet, the flagstaff, then $\angle$ BFC = 90°, BCA = 38′, and ACF = 14°; hence, from the two right-angled Δs ACF, BCF (where AF = h, and CF = d), we have—

$$h = d \tan 14^\circ \qquad (1)$$

$$h + 24 = d \tan 14^\circ\, 38' \qquad (2)$$

Dividing (2) by (1) we have—

$$1 + \frac{24}{h} = \frac{\tan 14^\circ\, 38'}{\tan 14^\circ}.$$

$$\left.\begin{array}{r} \log\tan 14^\circ\ 38' = 9.4168099 \\ \log\tan 14^\circ = 9.3967711 \end{array}\right\} \text{subtract}$$

$$\therefore \log\left(1 + \frac{24}{h}\right) = 0.0200388$$

$$\therefore 1 + \frac{24}{h} = 1.047222 \therefore \frac{24}{h} = .047222$$

$$\therefore h = \frac{24}{.047222} = \frac{24000000}{47222} = 508.2 \text{ feet.}$$

Also by (1),

$$d = \frac{h}{\tan 14^\circ} \therefore \log d = \log h - \log\tan 14^\circ + 10.$$

$$\left.\begin{array}{r} 10 + \log h = 10 + \log 508.2 = 12.7060347 \\ \log\tan 14^\circ = 9.3967711 \end{array}\right\} \text{subtract}$$

$$\therefore \log d = 3.3092636$$

$$\text{and} \therefore d = 2038.2 \text{ feet.}$$

(5.) (Fig., p. 70, Manual.)

Let C be the top of the mountain, DE the horizontal line through the base of the mountain, BE the pillar = 220 feet, BF = d, and CF = h; then $\angle$BCE = $1^\circ\ 12'$, and CBF = $12^\circ\ 20'$.

Also, let CF meet DE in G, then $\angle$CEG = CHG + BCE = $12^\circ\ 20' + 1^\circ\ 12' = 13^\circ\ 32'$; hence CG = EG tan CEG, and

CF = BF tan CBF, that is,

$$h + 220 = d\tan 13^\circ\ 32' \qquad (1)$$

$$h = d\tan 12^\circ\ 20' \qquad (2)$$

Dividing (1) by (2), we have—

$$1 + \frac{220}{h} = \frac{\tan 13^\circ\ 32'}{\tan 12^\circ\ 20'}$$

$$\left.\begin{array}{l}\log \tan 13^\circ 32' = 9.3814655 \\ \log \tan 12^\circ 20' = \underline{9.3397391}\end{array}\right\} \text{subtract}$$

$$\therefore \log\left(1 + \frac{220}{h}\right) = 0.0417264$$

$$\therefore 1 + \frac{220}{h} = 1.10084555 \therefore \frac{220}{h} = .10084555$$

$$\therefore h = \frac{\sqrt{220}}{.10084555} = \frac{22000000000}{10084555} = 2181.5 \text{ feet,}$$

the height of the mountain above the top of the pillar.

Again by (2),

$$d = \frac{h}{\tan 12^\circ 20'} \therefore \log d = \log h + 10 - \log \tan 12^\circ 20'$$

$$\left.\begin{array}{r}10 + \log h = 10 + \log 2181.5 = 13.3387552 \\ \log \tan 12^\circ 20' = \underline{9.3397391}\end{array}\right\} \text{subtract}$$

$$\therefore \log d = 3.9990161$$

$$\therefore d = 9977.4 \text{ feet.}$$

(6.) Here obviously (since 248 yards = 744 feet) height required = 744 sin $34^\circ 15' = 744 \times .5628049$

$$= 418.7268456 \text{ feet. } \quad \textit{Ans.}$$

(7.) Here distance required $= 490 \cot 13^\circ 49' = d$,

$$\therefore \log d = \log 490^\circ + \log \cot 13^\circ 49' - 10$$

$$\left.\begin{array}{r}\log 490 = 2.6901961 \\ \log \cot 13^\circ 49' - 10 = \underline{0.6091849}\end{array}\right\} \text{add}$$

$$\therefore \log d = 3.2993810$$

$$\therefore d = 1992.42 \text{ feet. } \quad \textit{Ans.}$$

(8.) Let CE (Fig., p. 69, Manual) be the pillar, DE the horizontal distance required $= d$, and AD $= 5$ feet, the spectator; then if AB be drawn parallel to DE, we have $\angle$ CAE $= 45^\circ$ by hypothesis; also CB $= 55$ feet, BE $= 5$ feet, and CE $= 60$ feet, hence

$$AC^2 = AB^2 + CB^2 = d^2 + 55^2 = d^2 + 3025$$

$$AE^2 = AB^2 + BE^2 = d^2 + 5^2 = d^2 + 25.$$

Now $CE^2 = AC^2 + AE^2 - 2AC \cdot AE \cos CAE$, that is,

$$60^2 = 3600 = 2d^2 + 3050 - 2\sqrt{\{(d^2 + 3025)(d^2 + 25)\}}$$
$$\times \cos 45^\circ,$$

$\therefore 550 = 2d^2 - \sqrt{\{2(d^2 + 3025)(d^2 + 25)\}}$, since $\cos 45^\circ$

$$= \frac{1}{\sqrt{2}} \text{ and } 2 \times \frac{1}{\sqrt{2}} = \sqrt{2};$$

$$\therefore 2d^2 - 550 = \sqrt{\{2(d^2 + 3025)(d^2 + 25)\}}$$

$$\therefore 4d^4 - 2200\, d^2 + 302500 = 2(d^4 + 3050\, d^2 + 75625)$$

$$= 2d^4 + 6100\, d^2 + 151200$$

$$\therefore d^4 - 4150\, d^2 = -\, 75650$$

$$\therefore d^2 = 2075 \pm \sqrt{(2075^2 - 75650)}$$

$$= 2075 \pm \sqrt{(4305625 - 75650)}$$

$$= 2075 \pm \sqrt{(4229975)} = 2075 \pm 2056.69;$$

$\therefore d^2 = 4131.69$ (or 18.31, which, it is easy to see, must be rejected),

$$\therefore d = \sqrt{(4131.69)} = 64.27 \text{ feet.} \quad \textit{Ans.}$$

(9.)

$$
\begin{array}{rl}
 & 2)10600 \\
 & \underline{5300} \\
\text{Extract} \;|\; & 15900 \quad (126 \text{ miles. } \textit{Ans.} \\
\text{sq. root.} \;|\; & \underline{144} \\
246) & 1500 \\
 & \underline{1476} \\
 & 24
\end{array}
$$

(10.)

$$
\begin{array}{r}
2)210 \\
\underline{105} \\
315 \quad (17.748 \text{ miles (statute)}. \\
\underline{1} \\
27)215 \\
\underline{189} \\
347)2600 \\
\underline{2429} \\
3544)17100 \\
\underline{14176} \\
35488)292400 \\
\underline{283904}
\end{array}
$$

Since a statute mile contains 5280 feet, and a nautical mile 6076 feet, we have—

$$6076 : 5280 :: 17.748 \text{ miles} : x \text{ (the answer)};$$

$$\therefore x = \frac{5280 \times 17.748}{6076} = \frac{93709.44}{6076} = 15.42. \quad \textit{Ans.}$$

(11.)

$$
\begin{array}{r}
2)600 \\
\underline{300}
\end{array}
$$

$$\therefore \sqrt{(900)} = 30 \text{ miles.} \quad \textit{Ans.}$$

(12.) Here dip $= \sqrt{(80)} = 8'.94$. *Ans.*

(13.) Let δ represent the dip expressed in minutes, and d the distance of the sea horizon in miles; then, since the radius of Earth = 3956 miles, and an arc = radius subtends at centre an $\angle$ of $206265''$ $= \frac{206265}{60}$ minutes, we have for circular measure of d the expression $\frac{d}{3956}$, and if we multiply this by $\frac{206265}{60}$, we obtain δ in minutes, that is—

$$\frac{d}{3956} \times \frac{206265}{60} = \delta$$

$$\therefore d = \frac{60 \times 3956}{206265} \times \delta = \frac{12 \times 3956}{41253} \times \delta = \frac{4 \times 3956}{13751} \times \delta$$

$$= \frac{15824}{13751} \times \delta = 1.1507 \times \delta. \quad \textit{Ans.}$$

(14.) Since atmospheric refraction is neglected, we have dip in minutes $= 1.06 \sqrt{(h)} = 1.06 \sqrt{(12170)}$ $= 1.06 \times 110.317 = 116'.93602 = 1^\circ\ 57'$, very nearly; hence, by answer to preceding question—

Distance in miles $= 1.15 \times 117 = 134.55$, or 135 miles nearly.

(15.) (Fig., p. 70, Manual.) Let CF represent the house, AB = 50 feet, the measured base, then $\angle$ CBF $= 36^\circ\ 14'$, $\angle$ CAF $= 25^\circ\ 10'$; also let CF $= h$, and BF $= d$.

From the right-angled Δ^s CBF, CAF we have

$$\text{CF} = \text{BF} \tan \text{CBF}, \text{ or } h = d \tan 36^\circ\ 14', \qquad (1)$$

and

$$\text{CF} = \text{AF} \tan \text{CAF}, \text{ or } h = (d + 50) \tan 25^\circ\ 10', \quad (2)$$

$$\therefore (d + 50) \tan 25^\circ\ 10' = d \tan 36^\circ\ 14',$$

or,
$$1 + \frac{50}{d} = \frac{\tan 36^\circ 14'}{\tan 25^\circ 10'}$$

$$\left.\begin{array}{l}\log \tan 36^\circ 14' = 9.8649755 \\ \log \tan 25^\circ 10' = 9.6719628\end{array}\right\} \text{subtract}$$

$$\therefore \log\left(1 + \frac{50}{d}\right) = 0.1930127$$

$$\therefore 1 + \frac{50}{d} = 1.5596 \therefore \frac{50}{d} = .5596, \text{ and } d = \frac{50}{.5596}$$

$$= \frac{500000}{5596} = 89.35 \text{ feet, the distance required.}$$

Again by (1) we have—

$$h = d \tan 36^\circ 14' = 89.35 \times \tan 36^\circ 14'$$

$$\therefore \log h = \log 89.35 + \log \tan 36^\circ 14' - 10$$

$$= 1.9510946 + 9.8649755 - 10 = 1.8160701$$

$$\therefore h = 65.474 \text{ feet, the height required.}$$

(16.) Let the $\angle ACO = \phi$, and $\therefore BCO = 180^\circ - \phi$.

Hence,
$$\frac{\sin A}{\sin \alpha} = \frac{CO}{AC}$$

$$\frac{\sin B}{\sin \beta} = \frac{CO}{BC}$$

$\therefore \dfrac{\sin A}{\sin \alpha} = \dfrac{\sin B}{\sin \beta}$, since $AC = BC$ by the hypothesis,

or,
$$\frac{\sin A}{\sin B} = \frac{\sin \alpha}{\sin \beta} \quad (1)$$

Again,
$$A = 180° - (\phi + a)$$
$$B = \phi - \beta$$
$$\therefore \frac{\sin A}{\sin B} = \frac{\sin(\phi + a)}{\sin(\phi - \beta)} \qquad (2)$$

From (1) and (2) we have—
$$\frac{\sin(\phi + a)}{\sin(\phi - \beta)} = \frac{\sin a}{\sin \beta}$$

or,
$$\frac{\sin\phi \cos a + \cos\phi \sin a}{\sin\phi \cos\beta - \cos\phi \sin\beta} = \frac{\sin a}{\sin\beta}$$

or,
$$\frac{\cos a + \cot\phi \sin a}{\cos\beta - \cot\phi \sin\beta} = \frac{\sin a}{\sin\beta}$$

$$\therefore \cos a \sin\beta + \cot\phi \sin a \sin\beta = \sin a \cos\beta$$
$$- \cot\phi \sin a \sin\beta$$

$$\therefore 2 \cot\phi = \frac{\sin a \cos\beta - \cos a \sin\beta}{\sin a \sin\beta}$$
$$= \frac{\sin(a - \beta)}{\sin a \sin\beta}. \quad \textit{Ans.}$$

(17.) Here half the $\angle$ which the chord joining the two entrances (i. e. the required distance d) subtends at the centre of the $\odot$ is $34° 40'$, but this chord is plainly $d = 2r \sin 34° 40' = 80 \times .5688011 = 45.504088$ feet. *Ans.*

(18.) (Fig., p. 78, Manual.)

Let P be the place of the spectator, and A, C, B, the three pillars; then AC = CB = 40 feet, $\angle$ APC $= 31° 10' = a$, and $\angle$ BPC $= 42° 12' = \beta$, and it is required to find PA $= a$, PB $= b$, and PC $= c$. Put

$\angle$ ACP $= \phi$, then by the result of question 16 we have—

$$2 \cot \phi = \frac{\sin (\alpha - \beta)}{\sin \alpha \sin \beta} = \frac{\sin 11^\circ 2'}{\sin 31^\circ 10' \sin 42^\circ 12'}$$

$$\therefore \log \cot \phi = \log \sin 11^\circ 2' - \log \sin 31^\circ 10'$$
$$- \log \sin 42^\circ 12' - \log 2 + 20$$

$$\left.\begin{aligned} \log \sin 31^\circ 10' &= 9.7139349 \\ \log \sin 42^\circ 12' &= 9.8271887 \\ \log 2 &= 0.3010300 \end{aligned}\right\} \text{add}$$

$$\left.\begin{aligned} &19.8421536 \\ \log \sin 10^\circ 2' + 20 = &29.2818967 \end{aligned}\right\} \begin{array}{l}\text{Subtract the upper} \\ \text{from the lower.}\end{array}$$

$$\therefore \log \cot \phi = 9.4397431$$

$$\therefore \phi = 74^\circ 36' 36'', \text{ or } 180^\circ - 74^\circ 36' 36''$$
$$= 105^\circ 23' 24'',$$

which latter value we shall employ in the future investigation of this problem.

$$\text{Hence the } \angle \text{PAC} = 180^\circ - 105^\circ 23' 24'' - 31^\circ 10'$$
$$= 43^\circ 26' 36''$$

$$\therefore a = \text{AC} \cdot \frac{\sin \phi}{\sin a} = 40 \times \frac{\sin 74^\circ 36' 36''}{\sin 31^\circ 10'},$$

since $\sin \phi = \sin (180^\circ - \phi) = \sin 74^\circ 36' 36''$.

$$\left.\begin{aligned} \log 40 &= 1.6020600 \\ \log \sin 74^\circ 36' 36'' &= 9.9841409 \end{aligned}\right\} \text{add}$$

$$\left.\begin{aligned} &11.5862009 \\ \log \sin 31^\circ 10' = &\ 9.7139349 \end{aligned}\right\} \text{subtract}$$

$$\therefore \log a = 1.8722660$$

$$\therefore a = 74.518 \text{ feet.}$$

$$c = AC \cdot \frac{\sin PAC}{\sin a} = 40 \times \frac{\sin 43^\circ\ 26'\ 36''}{\sin 31^\circ\ 10'}$$

$$\left.\begin{array}{r} \log 40 = 1.6020600 \\ \log \sin 43^\circ\ 26'\ 36'' = 9.8373591 \end{array}\right\} \text{add}$$

$$\left.\begin{array}{r} 11.4394191 \\ \log \sin 31^\circ\ 10' = 9.7139349 \end{array}\right\} \text{subtract}$$

$$\therefore \log c = 1.7254842$$

$\therefore c = 53.147$ feet, which is the distance from the middle pillar.

$$b = BC \cdot \frac{\sin 74^\circ\ 36'\ 36''}{\sin 42^\circ\ 12'} = 40 \times \frac{\sin 74^\circ\ 36'\ 36''}{\sin 42^\circ\ 12'}$$

$$\left.\begin{array}{r} \log 40 = 1.6020600 \\ \log \sin 74^\circ\ 36'\ 36'' = 9.9841409 \end{array}\right\} \text{add}$$

$$\left.\begin{array}{r} 11.5862009 \\ \log \sin 42^\circ\ 12' = 9.8271887 \end{array}\right\} \text{subtract}$$

$$\therefore \log b = 1.7590122$$

$$\therefore b = 57.413 \text{ feet.}$$

(19.) N. B. The angles given in this question are incompatible with the other *data* of the problem; they should be $19^\circ\ 27'\ 11''$, and $16^\circ\ 7'\ 12''$ respectively, instead of $25^\circ\ 38'$ and $20^\circ\ 14'$. The *incorrect* values give the angle (ϕ) between the diagonals, $\phi = 72^\circ\ 32'\ 30''$, which is only a few seconds less than the true value. With consistent *data* the problem may be solved as follows:—

Let ABCD be the rectangular court, E the middle point of BC, and F of CD, the diagonal AC = BD = 75 feet, $\angle EAC = 19^\circ\ 27'\ 11'' = a$, $\angle FAC = 16^\circ\ 7'\ 12'' = \beta$, and if AC meet EF in G, and BD in O, and OH be perpendicular to AB in the point H, it is plain by

geometry that EG = GF, and $\angle$ EGA = BOA = ϕ (the angle between the diagonals of court), and that OH = $\frac{1}{2}$ of BC, and BH = HA, since BO = OA = $\frac{1}{2}$ of AC, and that $\angle$ BOH = $\angle$ AOH.

Now in the Δ EAF, since EF is bisected in G, we have, by the result of quesion 16,

$$2 \cot \phi = \frac{\sin(\alpha - \beta)}{\sin \alpha \sin \beta} = \frac{\sin 3^\circ\ 19'\ 59''}{\sin 19^\circ\ 27'\ 11''.\ \sin 16^\circ\ 7'\ 12''}$$

$$\therefore \log \cot \phi = \log \sin 3^\circ\ 19'\ 59'' + 20 - \log 2$$

$$- \log \sin 19^\circ\ 27'\ 11'' - \log \sin 16^\circ\ 7'\ 12''.$$

$$\left.\begin{aligned} \log 2 &= 0.3010300 \\ \log \sin 19^\circ\ 27'\ 11'' &= 9.5224890 \\ \log \sin 16^\circ\ 7'\ 12'' &= \underline{9.4434977} \end{aligned}\right\} \text{add}$$

$$\left.\begin{aligned} &19.2670167 \\ \log \sin 3^\circ\ 19'\ 59'' + 20 = &\ \underline{28.7644751} \end{aligned}\right\} \begin{array}{l}\text{Take upper} \\ \text{from lower.}\end{array}$$

$$\therefore \log \cot \phi = 9.4974584$$

$$\therefore \phi = 72^\circ\ 32'\ 52''.$$

In the Δ BOH, right-angled at H, we now have—

BO = $\frac{1}{2}$ of 75 = 37.5 feet, and $\angle$ BOH = $\frac{1}{2}\,\phi$ = 36° 16′ 26″;

hence, BH = BO sin BOH = 37.5 sin 36° 16′ 26″,

and OH = BO cos BOH = 37.5 cos 36° 16′ 26″

$$\left.\begin{aligned} \log 37.5 &= 1.5740313 \\ \log \sin 36^\circ\ 16'\ 26'' &= \underline{9.7720618} \end{aligned}\right\} \begin{array}{l}\text{add, and re-} \\ \text{ject 10.}\end{array}$$

$$\therefore \log BH = 1.3460931$$

$\therefore$ BH = 22.1866 $\therefore$ 2 BH = 44.3732 feet, the shorter side of the court.

$$\left.\begin{array}{r}\log 37.5 = 1.5740313 \\ \log \cos 36^\circ\ 16'\ 26'' = 9.9064417\end{array}\right\} \text{ add, and reject 10.}$$

$$\therefore \log OH = 1.4804730$$

$$\therefore OH = 30.2324 \therefore 2\ OH = 60.4648 \text{ feet the longer side of the court.}$$

(20.) Let N be the Needles, NW the east and west line, A St. Alban's Head, and S the ship's place after 3 hours' sail; then, since AS is due north and south, the $\angle$AWN is a right angle; also by the hypothesis the $\angle$ANW $= \frac{3}{4}$ of a point, and the $\angle$WNS $= 3$ points, and AN $= 18$ nautical miles. (N. B. The reader will easily draw the figure from the above description).

Hence,

$$AW = AN \sin ANW = 18 \sin \tfrac{3}{4} \text{ point.}$$

$$\left.\begin{array}{r}\log 18 = 1.25527 \\ \log \sin \tfrac{3}{4} \text{ point} = 9.16652\end{array}\right\} \text{ add, and reject 10.}$$

$$\therefore \log AW = 0.42179$$

$$\therefore AW = 2.64$$

$$WN = AN \cos ANW = 18 \cos \tfrac{3}{4} \text{ point.}$$

$$\left.\begin{array}{r}\log 18 = 1.25527 \\ \log \cos \tfrac{3}{4} \text{ point} = 9.99527\end{array}\right\} \text{ add, and reject 10.}$$

$$\therefore \log WN = 1.25054$$

$$\therefore WN = 17.8$$

$$WS = WN \tan WNS = WN \tan 3 \text{ points.}$$

$$\left.\begin{array}{r}\log WN = 1.25054 \\ \log \tan 3 \text{ points} = 9.82489\end{array}\right\} \text{ add, and reject 10.}$$

$$\therefore \log WS = 1.07543$$

$$\therefore WS = 11.897, \text{ or } 11.9 \text{ nearly.}$$

$$SN = \frac{WN}{\cos WNS} = \frac{WN}{\cos 3 \text{ points}}$$

$$\left.\begin{aligned} 10 + \log WN &= 11.25054 \\ \log \cos 3 \text{ points} &= 9.91984 \end{aligned}\right\} \text{subtract.}$$

$$\therefore \log SN = 1.33070$$

$$\therefore SN = 21.414 \text{ knots in three hours.}$$

$$\therefore \text{Rate} = 7 \text{ knots.}$$

Also ship's distance from St. Alban's Head,

$$= AS = AW + WS = 2.6 + 11.9 = 14.5 \text{ knots.}$$

(21.) Draw a north and south line NS, and an east and west line EW, meeting in O, which take for Start Point; then if we draw OP, inclined to OW, towards the north, at an ∠ of half a point, and suppose OP = 80 miles, P will be Start Point; also, if we draw OH = 60 miles, and inclined to OW, at an ∠ of $4\frac{1}{2}$ points, H will be the position of Cape La Hogue, and if PH meet OW in A, and OS in B, then the ∠ OBP will be the inclination of the course towards the north, and PH its length. (N. B. From the above description the reader will easily draw the requisite figure).

Hence, we have the

$$\angle AOP = \tfrac{1}{2} \text{ point.}$$

$$\angle AOH = 4\tfrac{1}{2} \text{ points.}$$

and $$\therefore \angle POH = 5 \text{ points.}$$

Also, OP = 80 miles and OH = 60 miles.

$$\therefore \angle BOH = 3\tfrac{1}{2} \text{ points and } \tfrac{1}{2}(OHP + OPH)$$

$$= 90^\circ - 2\tfrac{1}{2} \text{ points} = 8 \text{ points} - 2\tfrac{1}{2} \text{ points} = 5\tfrac{1}{2} \text{ points.}$$

Now in Δ OPH we have—

$$\frac{\tan \frac{1}{2}(OHP - OPH)}{\tan \frac{1}{2}(OHP + OPH)} = \frac{80 - 60}{80 + 60} = \frac{1}{7}$$

$\therefore \log \tan \frac{1}{2}(OHP - OPH) = \log \tan \frac{1}{2}(OHP + OPH)$

$- \log 7 = \log \tan 5\frac{1}{2} \text{ points} - \log 7 = 10.27204$

$- 0.84510 = 9.42694$

$\therefore \frac{1}{2}(OHP - OPH) = 14^\circ\ 58' = 1\frac{1}{4}$ points;

but $\frac{1}{2}(OHP + OPH) = . \quad . \quad . \quad . \ 5\frac{1}{2}$ points.

$\therefore OHP = 6\frac{3}{4}$ points, and $OPH = 4\frac{1}{4}$ points.

Now $OBP = OHP - BOH = 6\frac{3}{4}$ points $- 3\frac{1}{2}$ points $= 3\frac{1}{4}$ points, towards the north,

$\therefore$ the course is N. W. $\frac{3}{4}$ N. *Ans.*

(22.) To find the length of the course PH (see last question), we have in Δ HOP

$\frac{1}{2}(OHP - OPH) = 14^\circ\ 58'$

$\frac{1}{2}(OHP + OPH) = 5\frac{1}{2} \text{ points} = 61^\circ\ 52'\ 30''$

$\therefore OHP = 76^\circ\ 50'\ 30''$ and $OPH = 46^\circ\ 54'\ 30''$;

also, $POH = 5 \text{ points} = 56^\circ\ 15'$ and $PO = 80$, $HO = 60$.

Hence, $$\frac{PH}{PO} = \frac{\sin POH}{\sin PHO}$$

$$\therefore PH = PO . \frac{\sin POH}{\sin PHO} = 80 \times \frac{\sin 56^\circ\ 15'}{\sin 76^\circ\ 50'\ 30''}$$

$$\left.\begin{array}{r} \log 80 = 1.9030900 \\ \log \sin 56^\circ\ 15' = 9.9198464 \end{array}\right\} \text{add}$$

$$\left.\begin{array}{r} 11.8229364 \\ \log \sin 76^\circ\ 50'\ 30'' = 9.9884450 \end{array}\right\} \text{subtract}$$

$\therefore \log PH = 1.8344914$

$\therefore PH = 68.311$ miles. *Ans.*

(23.) Let NS the north and south line, and EW the east and west line, meet in L, which take for the Tuscar Light, and let C, G, and R, be Carnsore, Greenore, and Roslare Points, respectively, (G being situated on the side of CR next L). Put CL = x, GL = y, and RL = z; we are to find x, y, and z.

Hence we have given CG = GR = $5\frac{1}{2}$ miles, and CR = 10 miles;

$$\text{also, } \angle CLG = CLN - GLN = 82^\circ\ 27' - 32^\circ\ 20' = 50^\circ\ 7',$$

$$\angle GLR = GLN - RLN = 32^\circ\ 20' - 11^\circ\ 10' = 21^\circ\ 10',$$

$$\text{and } \angle CLR = 82^\circ\ 27' - 11^\circ\ 10' = 71^\circ\ 17'.$$

Let us first compute the $\angle^s$ of the Δ CRG, which we shall denote by C, G, and R.

Since CG = GR, we have—

$$\cos C = \cos R = \frac{\frac{1}{2} CR}{CG} = \frac{5}{5\frac{1}{2}} = \frac{10}{11}$$

$$= .9090909 \therefore C = R = 24^\circ\ 37'\ 12''$$

$$\text{and} \therefore G = 180^\circ - 49^\circ\ 14'\ 24'' = 130^\circ\ 45'\ 36''.$$

Now put $\angle$ GRL = θ, then since G = CLR + θ + GCL

$$\therefore GCL = G - CLR - \theta = 130^\circ\ 45'\ 36'' - 71^\circ\ 17'$$

$$- \theta = 59^\circ\ 28'\ 36'' - \theta.$$

Hence from Δ GRL, we have—

$$\frac{\sin \theta}{\sin 21^\circ\ 10'} = \frac{y}{5\frac{1}{2}} \qquad (1)$$

$$\text{and from } \Delta \text{ GCL, } \frac{\sin (59^\circ\ 28'\ 36'' - \theta)}{\sin 50^\circ\ 7'} = \frac{y}{5\frac{1}{2}} \qquad (2)$$

From (1) and (2) we have—

$$\frac{\sin\theta}{\sin 21^\circ\ 10'} = \frac{\sin(59^\circ\ 28'\ 36'' - \theta)}{\sin 50^\circ\ 7'};$$

or, $$\frac{\sin\theta}{\sin 21^\circ\ 10'} = \frac{\sin 59^\circ 28' 36'' \cos\theta - \cos 59^\circ 28' 36'' \sin\theta}{\sin 50^\circ\ 7'};$$

or, $$\frac{\tan\theta}{\sin 21^\circ\ 10'} = \frac{\sin 59^\circ\ 28'\ 36'' - \cos 59^\circ\ 28'\ 36''.\tan\theta}{\sin 50^\circ\ 7'};$$

or, $$\tan\theta.\sin 50^\circ\ 7' = \sin 21^\circ\ 10'.\sin 59^\circ\ 28'\ 36''$$
$$- \sin 21^\circ\ 10' \cos 59^\circ\ 28'\ 36''.\tan\theta$$

$$\therefore \tan\theta = \frac{\sin 21^\circ\ 10'.\sin 59^\circ\ 28'\ 36''}{\sin 50^\circ\ 7' + \sin 21^\circ\ 10' \cos 59^\circ\ 28'\ 36''}$$

$$\left.\begin{aligned} \log\sin 21^\circ\ 10' - 10 &= \bar{1}.5576060 \\ \log\cos 59^\circ\ 28'\ 36'' - 10 &= \bar{1}.7057691 \end{aligned}\right\}\text{ add}$$

$$\therefore \log(\sin 21^\circ\ 10' \cos 59^\circ 28'\ 36'') = \bar{1}.2633751$$

$$\left.\begin{aligned} \therefore \sin 21^\circ\ 10' \cos 59^\circ\ 28'\ 30'' &= 0.18339 \\ \text{Also N. }\sin 50^\circ\ 7' &= 0.76735 \end{aligned}\right\}\text{ add}$$

$$0.95074$$

$$\therefore \log\tan\theta = \log\sin 21^\circ\ 10' + \log\sin 59^\circ\ 28'\ 36''$$
$$- (\log .95074 + 10)$$

$$\left.\begin{aligned} \log\sin 21^\circ\ 10' &= 9.5576060 \\ \log\sin 59^\circ\ 28'\ 36'' &= 9.9352162 \end{aligned}\right\}\text{ add}$$

$$\left.\begin{aligned} &19.4928222 \\ \log .95074 + 10 = &\ 9.9780618 \end{aligned}\right\}\text{ subtract}$$

$$\therefore \log\tan\theta = 9.5147604$$

$$\therefore \theta = 18^\circ\ 6'\ 58''$$

Hence from (1) we have—

$$y = 5.5 \times \frac{\sin\theta}{\sin 21^\circ\ 10'} = 5.5 \times \frac{\sin 18^\circ\ 6'\ 58''}{\sin 21^\circ\ 10'}$$

$$\left.\begin{aligned} \log 5.5 &= 0.7403627 \\ \log\sin 18^\circ\ 6'\ 58'' &= 9.4926817 \end{aligned}\right\} \text{add}$$

$$\left.\begin{aligned} & 10.2330444 \\ \log\sin 21^\circ\ 10' = {} & 9.5576060 \end{aligned}\right\} \text{subtract}$$

$$\therefore \log y = 0.6754384$$

$\therefore y = 4.73629$ miles, the distance from Greenore Point.

Again, from Δ GLR, we have—

$$\frac{z}{\text{GR}} = \frac{\sin(\theta + \text{GLR})}{\sin \text{GLR}}$$

$$\therefore z = 5.5 \times \frac{\sin(18^\circ\ 6'\ 58'' + 21^\circ\ 10')}{\sin 21^\circ\ 10'}$$

$$= 5.5 \times \frac{\sin 39^\circ\ 16'\ 58''}{\sin 21^\circ\ 10'}$$

$$\left.\begin{aligned} \log 5.5 &= 0.7403627 \\ \log\sin 39^\circ\ 16'\ 58'' &= 9.8013510 \end{aligned}\right\} \text{add}$$

$$\left.\begin{aligned} & 10.5417137 \\ \log\sin 21^\circ\ 10' = {} & 9.5576060 \end{aligned}\right\} \text{subtract}$$

$$\therefore \log z = 0.9841077$$

$\therefore z = 9.64068$ miles, the distance from Roslare Point.

Lastly, from the Δ CLG, we have—

$$\frac{x}{\text{CG}} = \frac{\sin(\text{CLG} + \text{GCL})}{\sin \text{CLG}}$$

$$= \frac{\sin(50^\circ\ 7' + 59^\circ\ 28'\ 36'' - 18^\circ\ 6'\ 58'')}{\sin 50^\circ\ 7'}$$

$$= \frac{\sin 91^\circ\ 28'\ 38''}{\sin 50^\circ\ 7'} = \frac{\sin 88^\circ\ 31'\ 22''}{\sin 50^\circ\ 7'}$$

$$\therefore x = 5.5 \times \frac{\sin 88^\circ\ 31'\ 22''}{\sin 50^\circ\ 7'}$$

$$\left.\begin{aligned} \log 5.5 &= 0.7403627 \\ \log \sin 88^\circ\ 31'\ 22'' &= 9.9998556 \end{aligned}\right\} \text{add}$$

$$\left.\begin{aligned} &10.7402183 \\ \log \sin 50^\circ\ 7' = {} &\ 9.8849945 \end{aligned}\right\} \text{subtract}$$

$$\therefore \log x = 0.8552238$$

$\therefore x = 7.165125$ miles, the distance from Carnsore Point.

(24.) (Fig., page 70, Manual.)

Let CF represent the height of the kite C, and A and B the two points of observation; then we have—

$$FC = AB \times \frac{\sin A \sin B}{\sin (B - A)} = 400 \times \frac{\sin 22^\circ\ 15' . \sin 48^\circ\ 22'}{\sin 26^\circ\ 7'}$$

$$\left.\begin{aligned} \log 400 &= 2.6020600 \\ \log \sin 22^\circ\ 15' &= 9.5782364 \\ \log \sin 48^\circ\ 22' &= 9.8735599 \end{aligned}\right\} \text{add}$$

$$\left.\begin{aligned} &22.0538563 \\ 10 + \log \sin 26^\circ\ 7' = {} &19.6436504 \end{aligned}\right\} \text{subtract}$$

$$\therefore \log FC = 2.4102059$$

$$\therefore FC = 257.16 \text{ feet. } \textit{Ans.}$$

(25.) From the formula in page 79 of the Manual, we have, making $a = b$ and $c = 2a$,

$$h^2 = \frac{a^3}{a \cot^2 A - 2a \cot^2 C + a \cot^2 B}$$

$$= \frac{a^2}{\cot^2 A - 2 \cot^2 C + \cot^2 B}$$

$$= \frac{250^2}{\cot^2 50^\circ 44' - 2 \cot^2 53^\circ 8' + \cot^2 48^\circ 21'}$$

$\log \cot^2 50^\circ 44' = 2 \log \cot 50^\circ 44' = 2 \times 9.9124981$

$= 19.8249962 = \bar{1}.8249962$ in common (*not* tabular) logarithms.

$\therefore \cot^2 50^\circ 44' = 0.668338$

$\log (2 \cot^2 53^\circ 8') = \log 2 + 2 \log \cot 53^\circ 8'$

$= 0.3010300 + 2 \times 9.8750102 = .3010300 + 19.7500204$

$= 20.0510504 = 0.0510504$ in common logarithms,

$\therefore 2 \cot^2 53^\circ 8' = 1.124745$

$\log \cot^2 48^\circ 21' = 2 \log \cot 48^\circ 21' = 2 \times 9.9490987$

$= 19.8981974 = \bar{1}.8981974$ in common logarithms.

$\therefore \cot^2 48^\circ 21' = 0.791038$ } add

but $\cot^2 50^\circ 44' = 0.668338$

1.459376 } subtract

and $2 \cot^2 53^\circ 8' = 1.124745$

0.334631 which is the denominator of h^2.

Hence, $h^2 = \dfrac{250^2}{0.334631}$ and $\therefore h = \dfrac{250}{\sqrt{(.334631)}}$

$$= \frac{250}{.5783} = 432.3 \text{ feet.} \quad \textit{Ans.}$$

(26.) Let A be the top of the ship's mast, B the point of the water line vertically below A, and C the hull of the second ship; then the distance BC, on the earth's surface, may be regarded as a right line perpendicular to AB, and the $\angle$BAC will be the complement of $14^\circ 34'$, and $\therefore = 90^\circ - 14^\circ 34' = 75^\circ 26'$; hence, we have BC = AB tan BAC = 86 tan $75^\circ 26'$.

$$\left.\begin{aligned} \log 86 &= 1.9344985 \\ \log \tan 75^\circ 26' - 10 &= 0.5852617 \end{aligned}\right\} \text{add}$$

$$\therefore \log BC = 2.5197602$$

$$\therefore BC = 330.94 \text{ feet. } \textit{Ans.}$$

(27.) The dip in minutes $= \sqrt{(86)} = 9'$ (see rule at foot of page 83, Manual); hence, when the dip is taken into account the $\angle$BAC is $= 75^\circ 26' - 9' = 75^\circ 17'$, and $\therefore$ BC = AB tan BAC becomes—

$$BC = 86 \tan 75^\circ 17'$$

$$\left.\begin{aligned} \log 86 &= 1.9344985 \\ \log \tan 75^\circ 17' - 10 &= 0.5806126 \end{aligned}\right\} \text{add}$$

$$\therefore \log BC = 2.5151111$$

$$\therefore BC = 327.42 \text{ feet. } \textit{Ans.}$$

(28.) Draw the north and south, east and west lines, NS and EW through Y, the position of the yacht at the beginning of the first course, and let A be her position at the beginning of the second course, and H the harbour. Let HA meet EW in B, then the $\angle$AYS = 6 points, HYS = 1 point, and $\therefore$ HYA = 7 points; also, YBA = 3 points, and $\therefore$ YAH = SYA + YBA = 5 points; hence YHA = 4 points, since $\angle$s of Δ AYH = 16 points, and YH = 5.8 miles; hence,—

$$\frac{YA}{YH} = \frac{\sin 4 \text{ points}}{\sin 5 \text{ points}}$$

$$\therefore YA = 5.8 \times \frac{\sin 4 \text{ points}}{\sin 5 \text{ points}}$$

$$\left.\begin{array}{r} \log 5.8 = 0.76343 \\ \log \sin 4 \text{ points} = 9.84948 \end{array}\right\} \text{add}$$

$$\left.\begin{array}{r} 10.61291 \\ \log \sin 5 \text{ points} = 9.91984 \end{array}\right\} \text{subtract}$$

$$\therefore \log YA = 0.69307$$

$$\therefore YA = 4.9325 \text{ miles the first course.}$$

Again, $\frac{HA}{YH} = \frac{\sin 7 \text{ points}}{\sin 5 \text{ points}} \therefore HA = 5.8 \times \frac{\sin 7 \text{ points}}{\sin 5 \text{ points}}$

$$\left.\begin{array}{r} \log 5.8 = 0.76343 \\ \log \sin 7 \text{ points} = 9.99157 \end{array}\right\} \text{add}$$

$$\left.\begin{array}{r} 10.75500 \\ \log \sin 5 \text{ points} = 9.91984 \end{array}\right\} \text{subtract}$$

$$\therefore \log HA = 0.83516$$

$$\therefore HA = 6.8416 \text{ miles, the second course.}$$

The sum of the two courses $= 4.9325 + 6.8416 = 11.7741$ miles which, divided by 7 *miles*, the rate per hour, gives 1.682 hours = 1 hour 41 minutes, the whole time of the voyage.

(29.) (Fig., p. 74, Manual).

Let C represent the buoy, then we have AB $= 1\frac{1}{2}$ mile $= 2640$ yards, $\angle A = 54^\circ\ 32'$, $\angle B = 39^\circ\ 15'$, and $\therefore \angle C = 180^\circ - 54^\circ\ 32' - 39^\circ\ 15' = 86^\circ\ 13'$.

$$\text{Hence, } AC = AB \cdot \frac{\sin 39^\circ\ 15'}{\sin 86^\circ\ 13'} = 2640 \times \frac{\sin 39^\circ\ 15'}{\sin 86^\circ\ 13'}$$

$$\text{and} \quad BC = AB \cdot \frac{\sin 54^\circ\ 32'}{\sin 86^\circ\ 13'} = 2640 \times \frac{\sin 54^\circ\ 32'}{\sin 86^\circ\ 13'}$$

$$\left.\begin{aligned} \log 2640 &= 3.4216039 \\ \log \sin 39^\circ\ 15' &= 9.8012015 \end{aligned}\right\} \text{add}$$

$$\left.\begin{aligned} &13.2228054 \\ \log \sin 86^\circ\ 15' = {} &9.9990529 \end{aligned}\right\} \text{subtract}$$

$$\therefore \log AC = 3.2237525$$

$\therefore$ AC = 1673.9 yards, the distance from A.

$$\left.\begin{aligned} \log 2640 &= 3.4216039 \\ \log \sin 54^\circ\ 32' &= 9.9108661 \end{aligned}\right\} \text{add}$$

$$\left.\begin{aligned} &13.3324700 \\ \log \sin 86^\circ\ 15' = {} &9.9990529 \end{aligned}\right\} \text{subtract}$$

$$\therefore \log BC = 3.3334171$$

$\therefore$ BC = 2154.8 yards, the distance from B.

(30.) If C be the position of the harbour, A and B the positions of the steamer and ship at the end of the $2\frac{1}{2}$ hours, and if AB meet the north and south line through C in S, then we have $\angle ACB = ACS + BCS = 1\frac{1}{4} + 5 = 6\frac{1}{4}$ points, $AC = 10\frac{1}{2} \times 2\frac{1}{2} = 26\frac{1}{4}$ knots $= b$, and $BC = 6 \times 2\frac{1}{2} = 15$ knots $= a$; hence,—

$$\angle B + A = 16 - 6\tfrac{1}{4} = 9\tfrac{3}{4} \text{ points} = 109^\circ\ 41'\ 15'',$$

$$\therefore \tfrac{1}{2}(B + A) = 54^\circ\ 50'\ 37''\tfrac{1}{2}.$$

$$\text{Now } \tan \tfrac{1}{2}(B - A) = \frac{b - a}{b + a} \cdot \tan \tfrac{1}{2}(B + A)$$

$$= \frac{26\frac{1}{4} - 15}{26\frac{1}{4} + 15} \tan 54^\circ\ 50'\ 37''\ \tfrac{1}{2}$$

$$= \frac{11\frac{1}{4}}{41\frac{1}{4}} \tan 54^\circ\ 50'\ 37''\ \tfrac{1}{2} = \frac{3}{11} \tan 54^\circ\ 50'\ 37''\ \tfrac{1}{2}$$

$$\left.\begin{array}{r} \log \tan 54^\circ\ 50'\ 37''\ \tfrac{1}{2} = 10.1522549 \\ \log 3 = \ \ 0.4771213 \end{array}\right\} \text{add}$$

$$\begin{array}{r} 10.6293762 \\ \log 11 = \ \ 1.0413927 \end{array}$$

$$\therefore \log \tan \tfrac{1}{2}(B - A) = 9.5879835$$

$$\therefore \tfrac{1}{2}(B - A) = 21^\circ\ 10'\ 6''\ \tfrac{1}{2}$$

$$\text{but } \tfrac{1}{2}(B + A) = 54^\circ\ 50'\ 37''\ \tfrac{1}{2}$$

$$\therefore B = 76^\circ\ 0'\ 44'' = 6\tfrac{3}{4} \text{ points,}$$

and $$A = 33^\circ\ 40'\ 31''.$$

$$\text{Again, } AB = BC\,\frac{\sin C}{\sin A} = 15 \times \frac{\sin 6\frac{1}{4} \text{ points}}{\sin 33^\circ\ 40'\ 31''}$$

$$\left.\begin{array}{r} \log 15 = 1.17609 \\ \log \sin 6\tfrac{1}{4} \text{ points} = 9.97384 \end{array}\right\} \text{add}$$

$$\left.\begin{array}{r} 11.14993 \\ \log \sin 33^\circ\ 40'\ 31'' = \ \ 9.74389 \end{array}\right\} \text{subtract}$$

$$\therefore \log AB = 1.40604$$

$\therefore$ AB = 25.47 miles, the required distance.

Hence from the above we have—

the $\angle$ CSB = $16 - 5 - 6\frac{3}{4} = 16 - 11\frac{3}{4} = 4\frac{1}{4}$ points.

$\therefore$ bearing is $4\frac{1}{4}$ points from north towards east,

that is, N. E. $\frac{1}{4}$ E.

(31.) If the line joining the summits, A and B, of the two mountains touch the earth in C, then AB is the required distance approximately; but if D be the foot of the mountain AD, and E the foot of BE, and we draw the diameters of the earth ADF and BEG, and put $2r = 7912$ miles for the diameter of the earth, we have by geometry—

$$AC^2 = FA \,.\, AD = 2r \,.\, AD, \text{ very nearly;}$$

$$BC^2 = BG \,.\, BE = 2r \,.\, BE, \text{ very nearly;}$$

$$\therefore AC = \sqrt{(7912 \times 3)} = \sqrt{(23736)} = 154 \text{ miles;}$$

$$BC = \sqrt{(7912 \times 2)} = \sqrt{(15824)} = 126 \text{ miles;}$$

$$\therefore AB = AC + BC = 154 + 126 = 280 \text{ miles.} \quad \textit{Ans.}$$

(32.) (Fig., p. 72, Manual.)

Let $AB = b = 58$ feet be the castle, and C the ship's hull, then by the formula, p. 73, Manual, we have—

$$\text{Distance FC} = h \times \frac{\cos d \cos d'}{\sin (d - d')}$$

$$= 58 \times \frac{\cos 5^\circ 47' \,.\, \cos 5^\circ 8'}{\sin 39'}$$

$$\left.\begin{array}{r} \log 58 = 1.7634280 \\ \log \cos 5^\circ 47' = 9.9977838 \\ \log \cos 5^\circ 8' = 9.9982546 \end{array}\right\} \text{add}$$

$$\left.\begin{array}{r} 21.7594664 \\ \log \sin 39' + 10 = 18.0547814 \end{array}\right\} \text{subtract}$$

$$\therefore \log FC = 3.7046850$$

$$\therefore FC = 5066.23 \text{ feet} = 1688.74 \text{ yards.} \quad \textit{Ans.}$$

(33.) Let C be the position of the tree, then it is plain that $\angle CAB = 124^\circ\ 4' - 60^\circ\ 33' = 63^\circ\ 31' = A$, and $\angle CBA = 57^\circ\ 56' - 0^\circ\ 28' = 57^\circ\ 28' = B$; hence $\angle C = 180^\circ - 63^\circ\ 31' - 57^\circ\ 28' = 59^\circ\ 1'$; also $AB = 250$ feet. If CD be perpendicular from C to AB, then CD is required breadth of river.

$$\text{Now } AC = AB \times \frac{\sin B}{\sin C}, \text{ and } CD = AC \sin A,$$

$$\therefore CD = AB \times \frac{\sin A \sin B}{\sin C}$$

$$= 250 \times \frac{\sin 63^\circ\ 31' . \sin 57^\circ\ 28'}{\sin 59^\circ\ 1'}$$

$$\left.\begin{aligned} \log 250 &= 2.3979400 \\ \log \sin 63^\circ\ 31' &= 9.9518541 \\ \log \sin 57^\circ\ 28' &= 9.9258681 \end{aligned}\right\} \text{add}$$

$$\left.\begin{aligned} &22.2756622 \\ \log \sin 59^\circ\ 1' + 10 = &19.9331415 \end{aligned}\right\} \text{subtract}$$

$$\therefore \log CD = 2.3425207$$

$$\therefore CD = 220.05 \text{ feet. } \textit{Ans.}$$

(34.) Here the dip of horizon in minutes

$$= \sqrt{(6700)} \text{ (see rule at foot of p. 83, Manual)}$$

$$= 82', \text{ nearly};$$

hence, if (Fig., p 78, Manual) we suppose D to be the balloon, $DP = 6700$ feet its height, and A to be St. Paul's, we shall have $\angle DAP = 10^\circ\ 33' + \text{dip} = 10^\circ\ 33' + 1^\circ\ 22' = 11^\circ\ 55'$;

$$\therefore \text{required distance} = AP = DP \cot DAP$$

$$= 6700 \times \cot 11^\circ\ 55'$$

$$\left.\begin{array}{r} \log 6700 = 3.8260748 \\ \log \cot 11^\circ 55' - 10 = 0.6756416 \end{array}\right\} \text{add}$$

$$\therefore \log AP = 4.5017164$$

$$\therefore AP = 31748 \text{ feet} = 6.0129 \text{ miles.} \quad \textit{Ans.}$$

(35.) (Fig., p. 76, Manual.)

Here $\angle CAD$ = difference of bearings of C and D from A $= 72^\circ - 51^\circ 30' = 20^\circ 30'$; since both bearings are E. of N. Since AC is $51^\circ 30'$ E. of N., $\therefore$ AC is $90^\circ - 51^\circ 30' = 38^\circ 30'$ N. of E., and B is $32^\circ 30'$ S. of E.; $\therefore \angle CAB = 38^\circ 30' + 32^\circ 30' = 71^\circ$; also, since AD lies 72° E. of N., it must lie $90^\circ - 72^\circ = 18^\circ$ N. of E.; and AB lies $32^\circ 30'$ S. of E.; $\angle DAB = 18^\circ + 32^\circ 30' = 50^\circ 30'$.

$\angle CBD = 22^\circ + 11^\circ 30' = 33^\circ 30'$, since C and D lie, one to E. of N., and the other to W. of N. with respect to B.

$$\angle ABD = 22^\circ + 57^\circ 30' = 79^\circ 30';$$

hence, $\angle ABC = ABD - CBD = 79^\circ 30' - 33^\circ 30' = 46^\circ$.

Hence by the proportion, p. 77, Problem VIII., we have CD (computed) : CD (given) :: 1000 : AB (required); that is, $598.35 : 2\frac{1}{4}$ miles :: 1000 : AB;

$$\therefore AB = \frac{2250}{598.35} = 3.76 \text{ miles.} \quad \textit{Ans.}$$

(36.) Make a right-angled isosceles Δ AEB, E being the right angle; draw AD meeting BE in D, and making the $\angle DAE = 15^\circ$, and BC meeting AE in C, and making the $\angle CBE = 15^\circ$; join CD. It will now be clear that if we suppose CD the breakwater, and A south of the extremity C, then B must be east of the extremity D; hence we have only to find CD when AB = 1250 yards.

Now $AE = \frac{1}{2}AB\sqrt{(2)} = 625\sqrt{(2)}$ since AE = EB, and $\angle E = 90^\circ$, and $CD = ED\sqrt{(2)}$.

But $\quad ED = AE \tan 15° = 625\sqrt{(2)} \times \tan 15°,$

$$\therefore CD = ED\sqrt{(2)} = 625\sqrt{(2)} \times \tan 15° \times \sqrt{(2)}$$
$$= 1250 \tan 15°.$$

$$\left.\begin{array}{r} \log 1250 = 3.0969100 \\ \log \tan 15° = \underline{9.4280525} \end{array}\right\} \text{add, and reject 10.}$$

$$\therefore \log CD = 2.5249625$$

$$\therefore CD = 334.937 \text{ yards. } \textit{Ans.}$$

(37.) Through C (Cork Harbour) draw the north and south line NS, and the east and west line EW; then draw CB, making an $\angle$ NCB towards the west, of $1\frac{3}{4}$ points; draw CA, making with CW towards the south an $\angle$ of 1 point, and make CA = 10; lastly, through A draw a north and south line AD, meeting CW in D, and make the $\angle$ DAB towards the east of AD = $2\frac{1}{4}$ points; then B is the position of the old Head of Kinsale, A of the ship at the end of the hour and quarter, and AB is the required distance.

Since the $\quad \angle ACD = 1$ point,

and ADC = 8 points (= 90°),

$\therefore \angle DAC = 7$ points, but DAB = $2\frac{1}{4}$ points,

$\therefore$ BAC = $7 - 2\frac{1}{4} = 4\frac{3}{4}$ points; also $\angle$ NCB = $1\frac{3}{4}$ points;

$\therefore$ BCA = $(8 - 1\frac{3}{4}) + 1 = 7\frac{1}{4}$ points; and

$\therefore$ ABC = $16 - 7\frac{1}{4} - 4\frac{3}{4} = 4$ points.

Hence, from Δ ABC we have—

$$AB = AC \cdot \frac{\sin BCA}{\sin ABC} = 10 \times \frac{\sin 7\frac{1}{4} \text{ points}}{\sin 4 \text{ points}}.$$

$$\left.\begin{array}{r} \log 10 = 1.00000 \\ \log \sin 7\tfrac{1}{4} \text{ points} = 9.99527 \end{array}\right\} \text{add}$$

$$\left.\begin{array}{r} 10.99527 \\ \log \sin 4 \text{ points} = \;\; 9.84948 \end{array}\right\} \text{subtract}$$

$$\therefore \log AB = 1.14579$$

$$\therefore AB = 13.989 \text{ miles.} \quad \textit{Ans.}$$

(38.) Draw CD cutting the east and west line EW in B, at an $\angle ABC$ below EW of $3\frac{1}{2}$ points, and take BA towards E = $10\frac{1}{2}$; draw AC towards the south of EW, and making the $\angle BAC$ = 3 points; make the right line CBD = 25; D and C will now represent Dover and Calais, and A the ship's place, and the $\angle DAB$ and distance DA are required.

In $\triangle$ ABC we have $\angle ABC = 3\frac{1}{2}$ points, BAC = 3 points, and $\therefore$ BCA = $16 - 3\frac{1}{2} - 3 = 9\frac{1}{2}$ points, and side AB = $10\frac{1}{2}$ miles; hence,

$$BC = AB \cdot \frac{\sin BAC}{\sin BCA} = 10.5 \times \frac{\sin 3 \text{ points}}{\sin 9\frac{1}{2} \text{ points}};$$

$$= 10.5 \times \frac{\sin 3 \text{ points}}{\sin 6\frac{1}{2} \text{ points}}$$

$$\left.\begin{array}{r} \log 10.5 = 1.02119 \\ \log \sin 3 \text{ points} = 9.74474 \end{array}\right\} \text{add}$$

$$\left.\begin{array}{r} 10.76593 \\ \log \sin 6\tfrac{1}{2} \text{ points} = \;\; 9.98088 \end{array}\right\} \text{subtract}$$

$$\therefore \log BC = 0.78505$$

$$\therefore BC = 6.0961.$$

Hence, $\quad DB = DC - BC = 25 - 6.0961$

$$= 18.9039 \text{ miles.}$$

Now in Δ DBA we have DB = 18.9039, AB = 10.5, and $\angle$ ABD = 2 right $\angle$s − ABC = 16 − $3\frac{1}{2}$ = $12\frac{1}{2}$ points.

$$\therefore \text{DAB} + \text{ADB} = 16 - 12\tfrac{1}{2} = 3\tfrac{1}{2} \text{ points.}$$

Hence, $\dfrac{\text{DB} - \text{BA}}{\text{DB} + \text{BA}} = \dfrac{\tan \frac{1}{2}(\text{DAB} - \text{ADB})}{\tan \frac{1}{2}(\text{DAB} + \text{ADB})}$;

that is, $\dfrac{18.9039 - 10.5}{18.9039 + 10.5} = \dfrac{\tan \frac{1}{2}(\text{DAB} - \text{ADB})}{\tan (\frac{1}{2} \times 3\frac{1}{2} \text{ points})}$;

$$\therefore \tan \tfrac{1}{2}(\text{DAB} - \text{ADB}) = \frac{8.4039}{29.4039}$$

$$\times \tan 1\tfrac{3}{4} \text{ points} = \frac{8.4039}{29.4039} \times \tan 19^\circ\, 41'\, 15'';$$

$$\left.\begin{array}{r} \log 8.4039 = 0.9244809 \\ \log \tan 19^\circ\, 41'\, 15'' = 9.5536472 \end{array}\right\} \text{add}$$

$$\left.\begin{array}{r} 10.4781281 \\ \log 29.4039 = \quad 1.4684048 \end{array}\right\} \text{subtract}$$

$$\therefore \log \tan \tfrac{1}{2}(\text{DAB} - \text{ADB}) = 9.0097233$$

$$\therefore \tfrac{1}{2}(\text{DAB} - \text{ADB}) = 5^\circ\, 50'\, 20''$$

but $$\tfrac{1}{2}(\text{DAB} + \text{ADB}) = 19^\circ\, 41'\, 15''$$

$$\therefore \text{DAB} = 25^\circ\, 31'\, 35''$$

and $$\text{ADB} = 13^\circ\, 50'\, 55''.$$

The complement of DAB = $90^\circ - 25^\circ\, 31'\, 35''$ = $64^\circ\, 28'\, 25''$ is the bearing of Dover W. of N., as required.

Again,

$$\frac{\text{AD}}{\text{AB}} = \frac{\sin \text{ABD}}{\sin \text{ADB}} = \frac{\sin 12\frac{1}{2} \text{ points}}{\sin 13^\circ\, 50'\, 55''} = \frac{\sin 3\frac{1}{2} \text{ points}}{\sin 13^\circ\, 50'\, 55''}$$

$$= \frac{\sin 39^\circ\, 22'\, 30''}{\sin 13^\circ\, 50'\, 55''}, \therefore \text{AD} = 10.5 \times \frac{\sin 39^\circ\, 22'\, 30''}{\sin 13^\circ\, 50'\, 55''};$$

$$\left.\begin{array}{r} \log 10.5 = 1.0211893 \\ \log \sin 39^\circ\ 22'\ 30'' = 9.8023585 \end{array}\right\} \text{add}$$

$$\left.\begin{array}{r} 10.8235478 \\ \log \sin 13^\circ\ 50'\ 55'' = 9.3790467 \end{array}\right\} \text{subtract}$$

$$\therefore \log AD = 1.4445011$$

$$\therefore AD = 27.8292 \text{ miles, the required distance.}$$

$$\therefore \angle ACB = 180^\circ - CAB - ABC = 180^\circ$$

$$- 71^\circ - 46^\circ = 63^\circ.$$

Now (see Problem VIII., p. 77, Manual), assume AB = 1000; hence, from Δ ACB we have—

$$AC = AB \times \frac{\sin ABC}{\sin ACB} = 1000 \times \frac{\sin 46^\circ}{\sin 63^\circ}.$$

$$\left.\begin{array}{r} \log 1000 = 3.0000000 \\ \log \sin 46^\circ = 9.8569341 \end{array}\right\} \text{add}$$

$$\left.\begin{array}{r} 12.8569341 \\ \log \sin 63^\circ = 9.9498809 \end{array}\right\} \text{subtract}$$

$$\therefore \log AC = 2.9070532$$

$$\therefore AC = 807.33.$$

Again in Δ ABD we have—

$$\angle DAB = 50^\circ\ 30',\ ABD = 79^\circ\ 30', \text{ and } \therefore \angle ADB$$

$$= 180^\circ - 50^\circ\ 30' - 79^\circ\ 30' = 50^\circ$$

$$\therefore AD = AB \times \frac{\sin ABD}{\sin ADB} = 1000 \times \frac{\sin 79^\circ\ 30'}{\sin 50^\circ}$$

$$\left.\begin{array}{r} \log 1000 = 3.0000000 \\ \log \sin 79^\circ 30' = \underline{9.9926661} \end{array}\right\} \text{add}$$

$$\left.\begin{array}{r} 12.9926661 \\ \log \sin 50^\circ = \underline{9.8842540} \end{array}\right\} \text{subtract}$$

$$\therefore \log AD = 3.1084121$$

$$\therefore AD = 1283.55$$

Hence in Δ CAD we now have—

$$AD = 1283.55 = a \text{ (suppose)}$$
$$AC = 807.33 = b,$$

and $\angle$ CAD $= 20^\circ 30' =$ C; to find CD $= c$ (suppose).

By the formula, p. 75, Manual, we have—

$$\cos \phi = \frac{2 \cos \frac{1}{2}C \sqrt{(ab)}}{a+b} \qquad (1)$$

and $$c = (a+b) \sin \phi. \qquad (2)$$

From (1) we have—

$$\log \cos \phi = \tfrac{1}{2} (\log a + \log b) + \log 2$$
$$+ \log \cos \tfrac{1}{2}C - \log (a+b)$$

$$\left.\begin{array}{r} \log a = 3.1084121 \\ \log b = \underline{2.9070532} \end{array}\right\} \text{add}$$

$$2)\underline{6.0154653}$$

$$\left.\begin{array}{r} 3.0077326 \\ \log 2 = 0.3010300 \\ \log \cos \tfrac{1}{2}C = \log \cos 10^\circ 15 = \underline{9.9930131} \end{array}\right\} \text{add}$$

$$\left.\begin{array}{r} 13.3017757 \\ \log (a+b) = \log 2090.88 = \underline{3.3203291} \end{array}\right\} \text{subtract}$$

$$\therefore \log \cos \phi = 9.9814466$$

$$\therefore \phi = 16^\circ 37' 44''.$$

From (2), $\log c = \log (a+b) + \log \sin \phi - 10$,

$$\begin{array}{rr} \log (a+b) = & 3.3203291 \\ \log \sin \phi = \log \sin 16^\circ\, 37'\, 44'' = & 9.4566263 \end{array} \Big\} \text{ add}$$

$$\begin{array}{r} 12.7769554 \\ 10. \end{array}$$

$$\therefore \log c = 2.7769554$$

$$\therefore c = 598.35 = \text{CD (computed).}$$

(39.) Let BD be the horizontal line, AB = 40 feet, the house, and CD = 180 feet, the tower; then the $\angle$ CAD = 36°, and BD = x is the required distance.

Draw AE perpendicular to CD, and put the $\angle$ DAE $= \theta$, and $\therefore$ CAE $= 36^\circ - \theta$.

From the two right-angled triangles DAE and CAE, we have—

$$40 = x \tan \theta \qquad (1)$$

$$140 = x \tan (36^\circ - \theta); \qquad (2)$$

$$\therefore \frac{\tan (36 - \theta)}{\tan \theta} = \frac{140}{40} = \frac{7}{2};$$

$$\therefore \frac{\tan (36^\circ - \theta) + \tan \theta}{\tan (36^\circ - \theta) - \tan \theta} = \frac{7+2}{7-2} = \frac{9}{5} = 1.8;$$

or, $$\frac{\sin (36^\circ - \theta) \cos \theta + \cos (36^\circ - \theta) \sin \theta}{\sin (36^\circ - \theta) \cos \theta - \cos (36^\circ - \theta) \sin \theta} = 1.8;$$

$$\therefore \frac{\sin 36^\circ}{\sin (36^\circ - 2\theta)} = 1.8$$

$$\therefore \sin (36^\circ - 2\theta) = \frac{\sin 36^\circ}{1.8}$$

$$\left.\begin{array}{r}\log\sin 36^\circ = 9.7692187\\ \log 1.8 = 0.2552725\end{array}\right\}\text{ subtract}$$

$$\therefore \log\sin(36^\circ - 2\theta) = 9.5139462$$

$$\therefore 36^\circ - 2\theta = 19^\circ\ 3'\ 34''$$

$$\therefore 2\theta = 36^\circ - 19^\circ\ 3'\ 34'' = 16^\circ\ 56'\ 26''$$

$$\therefore \theta = 8^\circ\ 28'\ 13''.$$

Hence from (1), $x = 40 \cot\theta = 40 \cot 8^\circ\ 28'\ 13''$;

$$\therefore \log x = \log 40 + \log\cot 8^\circ\ 28'\ 13'' - 10$$

$$= 1.6020600 + .8270451 = 2.4291051;$$

$$\therefore x = 268.6 \text{ feet.} \quad \textit{Ans.}$$

(40.) Through A, where the privateer lies, draw NS and EW, the north and south, east and west lines, and in the $\angle$NAE draw AB = 8, making the $\angle$BAE = 2 points, and through B draw BD direct south, and then draw BC, making the $\angle$ DBC = 5 points (now since ABD is the complement of BAE, which is = 2 points, ABD must = 6 points); take BC = $7 \times 2\frac{1}{2} = 17\frac{1}{2}$, and join AC; AC will be the distance run, and if BC meet AE in E, the $\angle$EAC will indicate the course.

Now in Δ ABC we have AB = 8 = c, BC = 17.5 = a, and $\angle$ABC = 6 + 5 = 11 points, $\therefore$ A + C = 16 − 11 = 5 points = $56^\circ\ 15'$; hence,

$$\frac{a-c}{a+c} = \frac{\tan\frac{1}{2}(A-C)}{\tan\frac{1}{2}(A+C)},$$

that is, $$\frac{17.5-8}{17.5+8} = \frac{\tan\frac{1}{2}(A-C)}{\tan 28^\circ\ 7'\ 30''};$$

$$\therefore \tan\tfrac{1}{2}(A-C) = \frac{9.5}{25.5} \times \tan 28^\circ\ 7'\ 30'' = \frac{19}{51}$$

$$\times \tan 28^\circ\ 7'\ 30'';$$

$$\left.\begin{aligned} \log \tan 28^\circ\, 7'\, 30'' &= 9.7279567 \\ \log 19 &= 1.2787536 \end{aligned}\right\} \text{add}$$

$$\left.\begin{aligned} & 11.0067103 \\ \log 51 &= 1.7075702 \end{aligned}\right\} \text{subtract}$$

$$\therefore \log \tan \tfrac{1}{2}(A - C) = 9.2991401$$

$$\therefore \tfrac{1}{2}(A - C) = 11^\circ\, 15'\, 43''$$

but $$\tfrac{1}{2}(A + C) = 28^\circ\, 7'\, 30'';$$

$$\therefore A = 39^\circ\, 23'\, 13'', \text{ and } C = 16^\circ\, 51'\, 47'' = 1\tfrac{1}{2} \text{ points.}$$

Hence the course is E. by S. $\frac{1}{2}$ S.

$$\text{Again, } AC = AB \times \frac{\sin B}{\sin C} = 8 \times \frac{\sin 11 \text{ points}}{\sin 16^\circ\, 51'\, 47''}$$

$$= 8 \times \frac{\sin 5 \text{ point}}{\sin 16^\circ\, 51'\, 47''} = 8 \times \frac{\sin 56^\circ\, 15'}{\sin 16^\circ\, 51'\, 47''}$$

$$\left.\begin{aligned} \log 8 &= 0.9030900 \\ \log \sin 56^\circ\, 15' &= 9.9198464 \end{aligned}\right\} \text{add}$$

$$\left.\begin{aligned} & 10.8229364 \\ \log \sin 16^\circ\, 51'\, 47'' &= 9.4625252 \end{aligned}\right\} \text{subtract}$$

$$\therefore \log AC = 1.3604112$$

$$\therefore AC = 22.93 \text{ miles, the required distance.}$$

Hence required rate $= 22.93 \div 2\frac{1}{2} = 9.172$ knots.

(41.) (Fig. 1, Key.) Let ABCD be the quadrilateral; a, b, c, and d, its sides, and $\angle$ BAD $= \alpha$, CDA $= \gamma$, and BGA $= \beta$. Draw BE and CF perpendicular to AD, and CH parallel to it.

Then $\quad AE = a \cos \alpha,$

$$EF = CH = b \cos \beta, \text{ since } \angle BCH = \beta,$$

and $\quad FD = c \cos \gamma;$

$$\therefore AE + EF + FD = AD = d = a \cos \alpha + b \cos \beta + c \cos \gamma \quad (1)$$

$$\therefore \text{by } (1) \; (d - b \cos \beta) \sin \alpha - b \sin \beta \cos \alpha$$

$$= (a \cos \alpha + c \cos \gamma) . \sin \alpha - b \sin \beta \cos \alpha = c \sin \alpha \cos \gamma$$
$$+ (a \sin \alpha - b \sin \beta) . \cos \alpha;$$

but $\quad a \sin \alpha = BE$, and $b \sin \beta = BH$;

$$\therefore a \sin \alpha - b \sin \beta = HE = CF = c \sin \gamma. \quad (2)$$

Hence, $(d - b \cos \beta) \sin \alpha - b \sin \beta \cos \alpha$

$$= c \sin \alpha \cos \gamma + c \sin \gamma \cos \alpha = c \sin (\alpha + \gamma),$$

which proves the third equation in the question.

Again, from (1) we have—

$$(d - b \cos \beta) \sin \gamma + b \sin \beta \cos \gamma = (a \cos \alpha + c \cos \gamma)$$
$$\sin \gamma + b \sin \beta \cos \gamma = a \cos \alpha \sin \gamma$$
$$+ (c \sin \gamma + b \sin \beta) \cos \gamma.$$

But by (2) we have $c \sin \gamma + b \sin \beta = a \sin \alpha$;

hence $\quad (d - b \cos \beta) \sin \gamma + b \sin \beta \cos \gamma$

$$= a \cos \alpha \sin \gamma + a \sin \alpha \cos \gamma = a \sin (\alpha + \gamma),$$

which proves the second equation in the question.

Lastly, from (1) we have—

$$a \cos \alpha + c \cos \gamma = d - b \cos \beta,$$

and from (2) $\quad a \sin \alpha - c \sin \gamma = b \sin \beta.$

Squaring these expressions, and adding, we have—

$$a^2 + c^2 + 2ac\,(\cos\alpha\cos\gamma - \sin\alpha\sin\gamma) = d^2 - 2bd\cos\beta + b^2;$$

or $$a^2 + c^2 + 2ac\cos(\alpha+\gamma) = d^2 + b^2 - 2db\cos\beta$$

$$\therefore \cos(\alpha+\gamma) = \frac{d^2 + b^2 - (a^2 + c^2 + 2db\cos\beta)}{2ac},$$

which proves the first equation in the question.

(42.) From equation (2) of last question we have—

$$\frac{a\sin(\alpha+\gamma)}{b\sin\beta} = \left(\frac{d}{b}\operatorname{cosec}\beta - \cot\beta\right)\sin\gamma + \cos\gamma$$

$$= \cot\phi\sin\gamma + \cos\gamma;$$

$$\therefore \frac{a\sin(\alpha+\gamma)\sin\phi}{b\sin\beta} = \cos\phi\sin\gamma + \cos\gamma\sin\phi = \sin(\gamma+\phi)$$

which proves the first equation.

Again from equation (3) of last question we have—

$$\frac{c\sin(\alpha+\gamma)}{b\sin\beta} = \left(\frac{d}{b}\operatorname{cosec}\beta - \cot\beta\right)\sin\alpha - \cos\alpha$$

$$= \cot\phi\sin\alpha - \cos\alpha$$

$$\therefore \frac{c\sin(\alpha+\gamma)\sin\phi}{b\sin\beta} = \cos\phi\sin\alpha - \cos\alpha\sin\phi = \sin(\alpha-\phi),$$

which proves the second equation.

(43.) $$\cos(\alpha+\gamma) = \frac{d^2 + b^2 - (a^2 + c^2 + 2db\cos\beta)}{2ac}$$

$$= \frac{100^2 + 63^2 - (70^2 + 44^2 + 200 \times 63\cos 19^\circ)}{2 \times 70 \times 44}$$

$$= \frac{10000 + 3969 - (4900 + 1936 + 12600 \cos 19^\circ)}{6160}$$

$$= \frac{7133 - 12600 \times .9455186}{6160} = \frac{7133 - 11913.53436}{6160}$$

$$= \frac{-4780.53436}{6160} = -.7760607;$$

$$\therefore \alpha + \gamma = 180^\circ - 39^\circ\,6' = 140^\circ\,54'.$$

Again, $\cot\phi = \dfrac{d}{b}\operatorname{cosec}\beta - \cot\beta = \dfrac{d}{b\sin\beta} - \cot\beta$

$$\left.\begin{aligned} \log b = \log 63 &= 1.7993405 \\ \log\sin\beta = \log\sin 19^\circ &= 9.5126419 \end{aligned}\right\}\text{add}$$

$$\left.\begin{aligned} &11.3119824 \\ \log d + 10 = \log 100 + 10 = &12.0000000 \end{aligned}\right\}\begin{array}{l}\text{Subtract the}\\ \text{upper from}\\ \text{the lower.}\end{array}$$

$$\therefore \log\frac{d}{b\sin\beta} = 0.6880176$$

$$\therefore \frac{d}{b\sin\beta} = 4.87548$$

$\cot\beta = \cot 19^\circ = 2.90421$ (by Table III., Manual);

$$\therefore \cot\phi = 1.97127 \text{ and } \therefore \phi = 26^\circ\,54'.$$

Now, $\sin(\alpha - \phi) = \dfrac{c\sin(\alpha+\gamma)\sin\phi}{b\sin\beta}$

$$\begin{aligned} \log c = \log 44 &= 1.6434527 \\ \log\sin(\alpha+\gamma) = \log\sin 39^\circ\,6' &= 9.7998062 \\ \log\sin\phi = \log\sin 26^\circ\,54' &= 9.6555559 \\ &\overline{21.0988148} \end{aligned}$$

(1)

$$\left.\begin{aligned} \log b = \log 63 &= 1.7993405 \\ \log \sin \beta = \log \sin 19^\circ &= \underline{9.5126419} \end{aligned}\right\} \text{add}$$

$$11.3119824 \qquad (2)$$

$$(1)—(2) \text{ gives } \log \sin (\alpha - \phi) = 9.7868324;$$

$\therefore\ \alpha - \phi = 37^\circ\ 44'$ (or $180^\circ - 37^\circ\ 44' = 142^\circ\ 16'$, which being greater than $\alpha + \gamma = 140^\circ\ 54''$, is inadmissible if negative values of α and γ be excluded); hence $\alpha = 37^\circ\ 44' + 26^\circ\ 54' = 64^\circ\ 38'$, and $\therefore$ since $\alpha + \gamma = 140^\circ\ 54'$ we have—

$$\gamma = 140^\circ\ 54' - 64^\circ\ 38' = 76^\circ\ 16'.$$

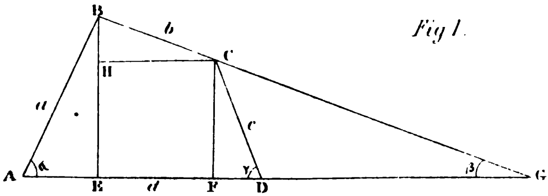

Fig 1.